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BUDGET ALLOCATION AND ENLISTMENT PREDICTION MODELS FOR THE NAVY--ETC(U)
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BUDGET ALLOCATION AND ENLISTMENT PREDICTION MODELS FOR THE NAVY'S RECRUITING COMMAND:

The Proper Balance Between Recruiter and Advertising Efforts.

9 Technical rept. no. 1, Jun 78-May 79,

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11 May ~~1~~ 1979

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1.0 INTRODUCTION AND SUMMARY

1.1 Overview

This effort, performed over the past year, has had the thrust of being able to model, predict and optimize the combined effects of advertising and recruiter efforts for the Navy's recruiting efforts. By being better able to simultaneously allocate and tradeoff both types of expenditures, both against themselves as well as geographically and over time, the Recruiting Command should be better able to meet their quality and quantity enlistment goals over time. The key product of this effort is a computer program to be run periodically by the Recruiting Command to help execute national and regional budgets and to be a credible tool in the budget building and budget defense process. By exercising the model in a "what-if" mode it can also be useful in exploring the potential impacts of various policy changes and help indicate where future field experimentation may be warranted.

1.2 Key Accomplishments

In order to accomplish the above, the efforts have had two concurrent thrusts:

1) to develop and validate response functions which would be capable of predicting for each of the regions of the country and by time (months) of the year

   i) the total number of contracts for males, enlisting for the first time, and

   ii) the total number of contracts by males who are enlisting for the first time and who are also (or will be) high school graduates.

In both cases these response functions would take as inputs the unique characteristics of that particular region, e.g. its QMA, number of high school graduates in the area, number of active NOIC leads in the region, amount of total advertising expenditures in the region of interest (both for the current period and for the previous months), number of recruiters in the region of interest (both currently and for the previous months), etc.

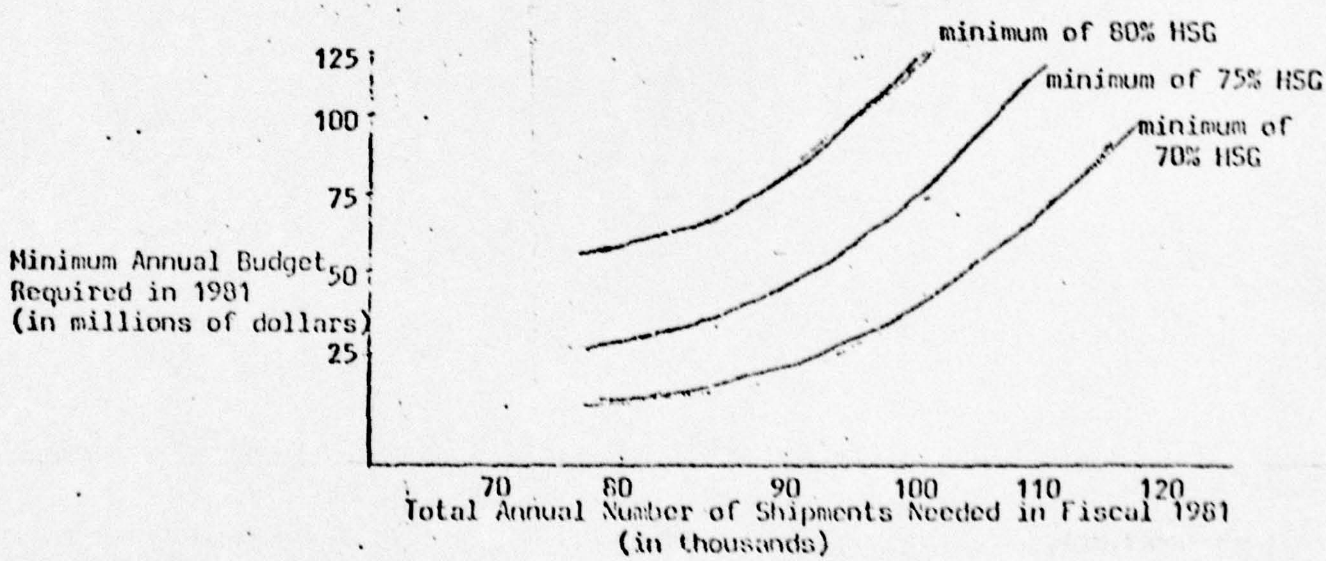
2) to develop and validate a budget allocation algorithm which would be capable of utilizing the above response functions to perform the following:

   i) In a budget request mode, to develop the minimum annual total recruiting budget needed for a given requirement of total enlistments (shipments) over the year with constraints on the minimum percent of these that must be HSG's. This program would take as inputs for each region of the country: the bank of NOIC leads available, the "stock" of advertising expenditures for the past several months, the number of recruiters in place for the past several months, demographic data about each region, and the amount of shipments already in the Delayed Entry Program.

   In this mode it would produce the optimal budget split between advertising and recruiting costs as well as the optimal budget allocations over time and by district. It is felt such a tool would be extremely helpful in the POM budget generation process, in the

Navy Controller reviews, and in the Presidential/ Congressional reviews.

The program could be run for a variety of quotas and goaling strategies, and their budget implications examined. The code developed, while sophisticated enough to solve the non-linear optimizations needed, (due to the non-linear response functions involved) also appears to run sufficiently fast (a few minutes of CPU time) so that it can help answer the difficult "what if" questions needed in a timely and defensible manner. In particular the computerized model could be used to draw up in advance of the decisions plots of the impact on the total dollars needed (and their optimal split) as a function of the yearly quota, their distribution over time, and the quality constraints. Hence one can develop curves of the type:



ii) <u>In a budget execution mode</u>, it can optimally

a) allocate a given amount of advertising dollars in terms of the geographic mix and timing,

b) allocate the optimal locations of recruiters for a fixed total number of recruiters,

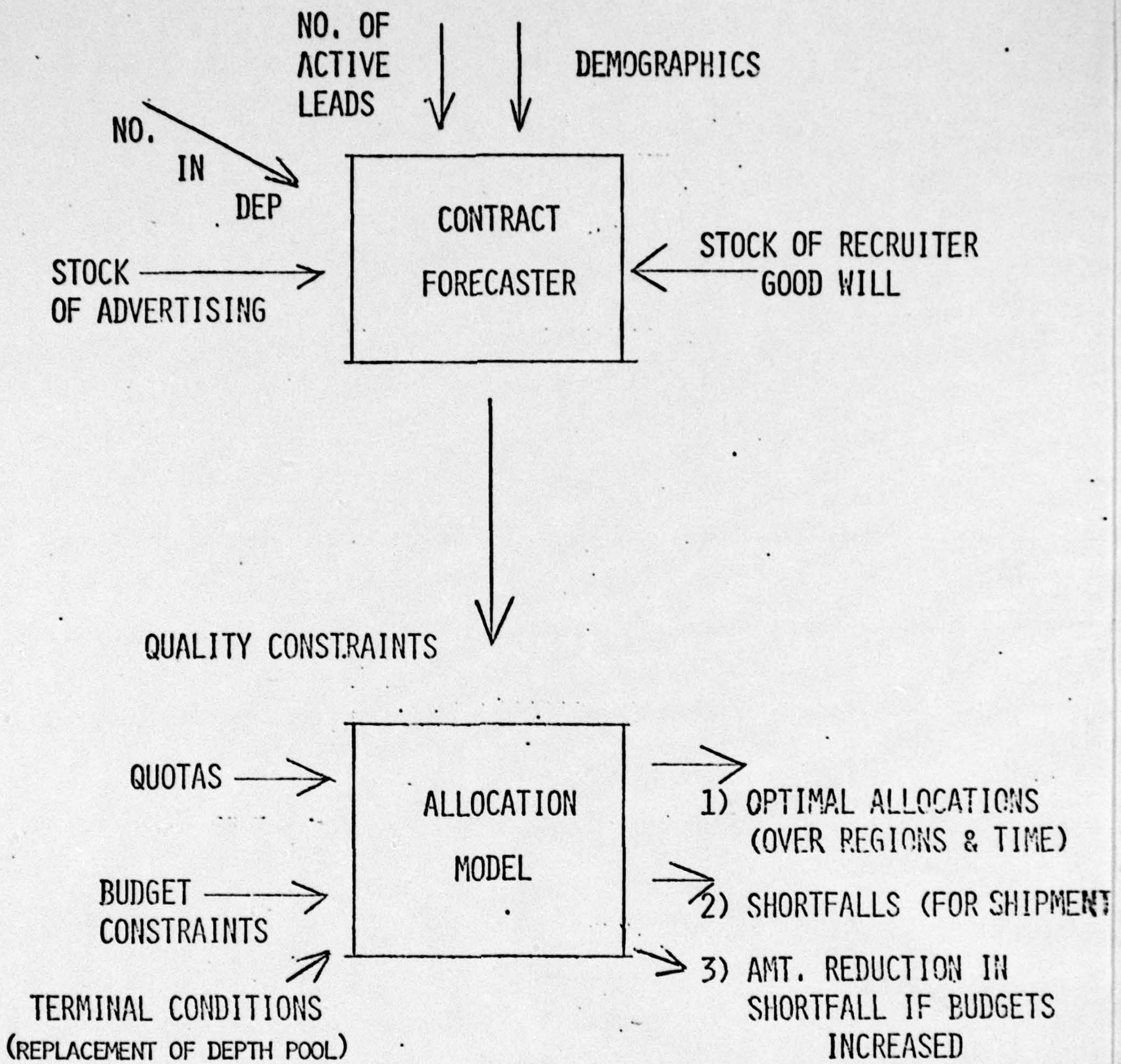
c) calculate the impacts, related to reducing any shipping quota shortfalls, of increasing either the advertising or recruiter budget by stated amounts, decreading quality constraints, decreasing minority quotas, etc.

d) it can eventually be used to give insights as to the proper split of advertising between the various models available, i.e. TV/radio, magazines, classified ads, direct mail, RAD materials, etc. as well as guidelines to recruiters on the proper targets for the size of the Delayed Entry Pool throughout the year. Flow diagrams depicting the interactions between products (1) and (2) follow.

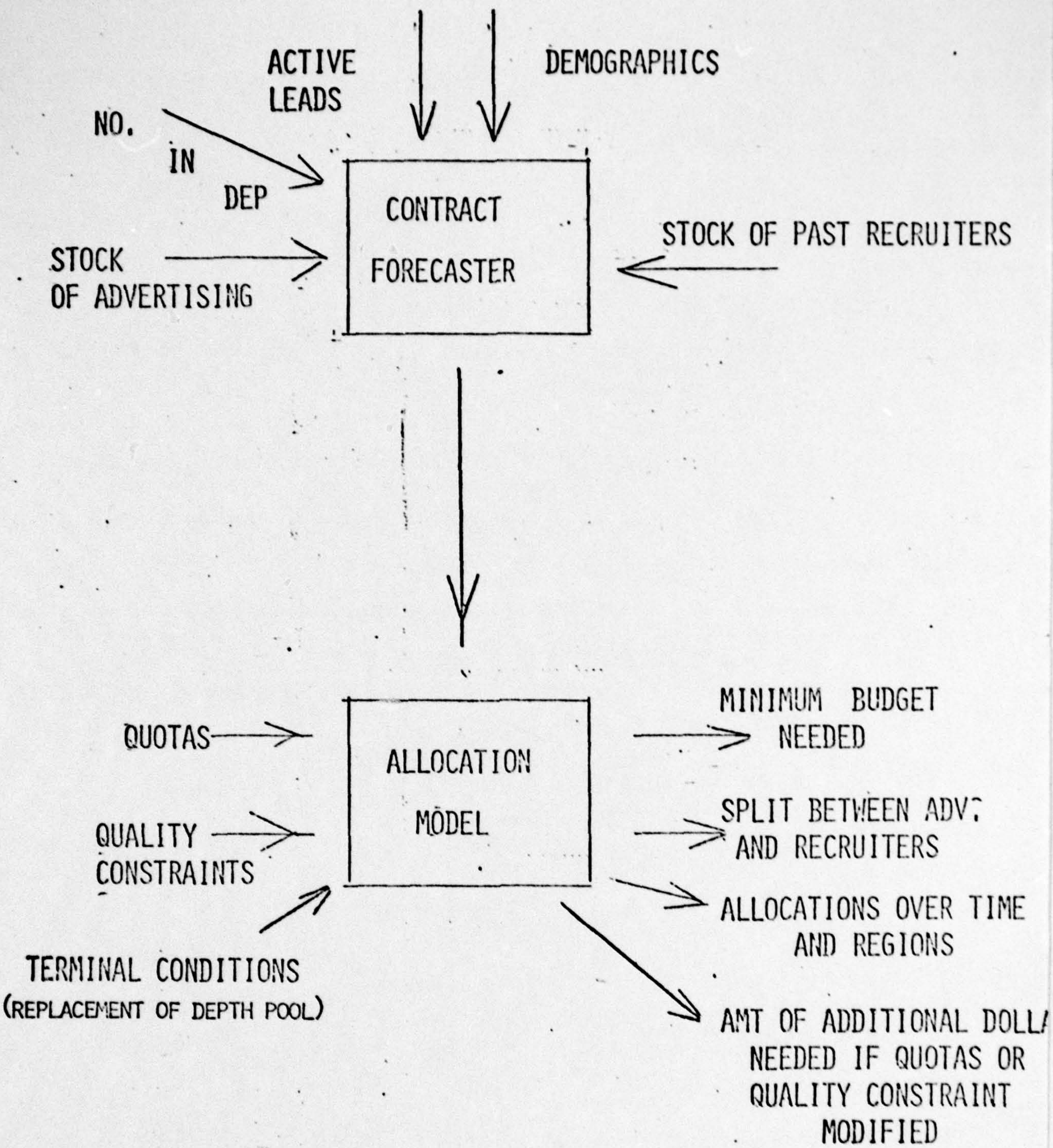
1.3 Organization of Remainder of Report

The remainder of the report is as follows: Section 2 describes the mathematical model used in the budget allocation part of the effort as well as several variations. Section 3 deals with the enlist-

OVERVIEW OF EXECUTION LOGIC



OVERVIEW OF BUDGET BUILDING LOGIC



ment prediction aspect of the work and its validation. Section 4 discusses the first of three runs made with the models. It deals with comparisons of actual and forecasted shipments for the actual regional/monthly expenditures made in CY77 (which deal with the descriptive ability of the allocation model). As one can see the model reproduces very well the number of total contracts for each of the twelve months, the number of HSG contracts for each of the twelve months, and the total number of shipments for each of the twelve months. This run helps instill some credibility and reasonableness with the procedure. The second run, the subject of Section 5, utilizes as budget constraints for the decision horizon the actual total expenditures experienced in CY77, together with a yearly shipping quota of 83,117 and a quality constraint (related to the percent of total contracts signed during the year who are HSG's) of 70%; the budget constraints were $71.37M for recruiters and their associated support, and $16.07M for advertising. In addition the model's terminal conditions were to have in place the same DEP pool that actually resulted at the end of CY77. The result of this run is that the quantity and quality quotas could be met, if the budget allocations were optimized (i.e. meet the quotas at minimum total costs), for about 6.8% less than was actually spent; further this would result in the mix of advertising expenditures, relative to recruiting efforts, changing from the 18% experienced in CY77 to about 13%. Given the limited accuracy of the contract prediction functions used (i.e. R^2's of .76), these results help lend confidence to the use of the model in generating and defending future budget requests, particularly if the contract predictor equations can be made more accurate.

Section 6 reports on the results of a sensitivity run in which the delay factors in the DEP program were treated as decision variables (in contrast to the other runs where they were inputed to the model using the empirical delays). The outputs from such exercises can be used to develop guidelines on the proper indepth goals throughout the year to best meet stated shipping quotas. Finally several appendices follow with supporting data.

2.0 BUDGET ALLOCATION

2.1 Definitions of Input Parameters

i) Let I denote the number of geographical units for which the allocation is desired. This would be 6 if regions are used as the decision unit or 43 if allocations are desired by district. In the results shown, I has the value of 6 since the allocation decisions were made on a regional basis.

ii) Let J denote the number of time periods in the horizon for which allocation decisions are desired; this of course would be 12 if one is considering monthly decisions for a year. It might be remarked that it is also possible to utilize the programs where the periods represent different lengths of time. This could be of advantage if a finer division of time were needed early in the horizon whereas a coarser time grid would be sufficient in the later stages.

iii) Let $\bar{J}$ denote the number of time periods in the total problem horizon which includes, in addition to the J periods in the decision horizon, a number of periods to incorporate the terminal conditions for the model. Since the Delayed Entry Program allows a maximum delay of 12 months from contract signing to shipment, a reasonable candidate for $\bar{J}$ is a year beyond J. (In the sample results $\bar{J}$ is 24 months.)

iv) Let K denote the number of classes recruits, e.g. high school graduates, non-high school graduates, Blacks, HSG's of mental Category I or II, women, etc. (In the runs made using the allocation program, two categories of recruits have been utilized, namely HSG's and non HSG's.

v) Let L_k denote the number of lagged periods in the enlistment production functions which relate contracts for the kth type of recruit to the dollar level of advertising and number of recruiters. Hence the number of recruiters and dollar level of advertising for up to L_k earlier periods is assumed to affect the number of contracts obtained in any given period. (In the runs made L_k equals 2; this value was chosen based on regression results using the Koyck distributed lag model).

vi) For any region let $e_{t,k}$ $(t=0,1,2\ldots,L_k;\ k=1,2,\ldots,K)$ denote the elasticity of the number of contracts, for the kth type of recruit signed in the current period, to the number of recruiters present t periods earlier. Hence a 1% change in the number of recruiters t periods earlier is assumed to produce a change of $e_{t,k}$ percent in the number of contracts for the kth type of recruit resulting in the current period. (From the regressions it was determined that the elasticities over all contracts were .29, .106, and .039, respectively for $t=0,1$, and 2; for high school graduate contracts, the elasticities were .391, .138 and .049. Hence since about 70% of all contracts are HSG's it appears recruiters are relatively more effective in obtaining enlistments of HSG's than in obtaining non-high school graduates.

vii) Let $f_{t,k}$ $(t=0,1,2,\ldots L_k;\ k=1,2,\ldots,K)$ denote the elasticity of the number of contracts, for the kth type of recruit signed in the current period, to the dollar level of total advertising (TV, magazine, classified ads, RAD, direct mail, etc.) expended t periods earlier. Hence a 1% change in the dollar level

of advertising impacting in a given region t periods earlier is related to a change of $f_{t,k}$ percent in the number of contracts for the kth type of recruit obtained in the current period. (From the regression results it was determined that the elasticities for total contracts are .125, .045 and .017 respectively for t=0,1 and 2, and for HSG contracts the elasticities are .081, .029, and .01; hence it appears that advertising plays an important role, especially for the non-high school graduate.

viii) Let $P_{i,j,k}$ ($i=1,2,\ldots I$; $j=1,2,\ldots J$; $k=1,2,\ldots K$) denote the constant term in the multiplicative enlistment response function which relates contracts, for the kth type of recruit obtained from the ith region in the jth period, to advertising and recruiter efforts in the L_k previous periods. Hence it reflects regional and seasonal differences related to the impact of advertising/recruiter efforts to obtaining contracts. Appendix A contains the $P_{i,j,k}$'s for total contracts, and for HSG graduates.

ix) Let $b_{j,v,k}$ ($j=1,2,\ldots,12$; $v=0,1,2,\ldots,12$; $k=1,2,\ldots,K$) denote the fraction of all contracts of type k, signed in period j, that ship v periods later. Note that in general $\sum_{v=0}^{12} b_{j,v,k}$ can be less than 1 since there is some attrition, i.e. some small percent of those that sign contracts in period j may change their mind and break their contract and hence never really enlist. (The 12x13 matrix of the $b_{j,v}$'s for k=1, i.e. total contracts, is included in Appendix B and was developed by the Recruiting Command from actual shipments by discerning

the elapsed time from signing of the contract to date of shipping; a similar effort is now underway to do the same for HSG's.

x) Let $a_{j,k}$ ($j=1,2,\ldots,J$; $k=1,2,\ldots,K$) denote the fraction of all contracts of type k, signed in period j that attrit. Hence $\sum_{v=0}^{13} b_{j,v,k} + a_{j,k} = 1$. (In the computer runs shown, $a_{j,k}$ is taken to be zero for all j's, since attrition factors for the DEP Program are not available. However a Data Collection effort has been requested. As a result there may be a small disparity between the computer model results and the actuals since no attrition is assumed.

xi) Let $Q_{j,k}$ ($j=1,2,\ldots,J,J+1,\ldots,\bar{J}$; $k=1,2,\ldots,K$) denote the quotas for shipments for the kth type of recruit in the jth period. Note that there are quotas for periods beyond the decision horizon of J which reflect the terminal conditions in the allocation model. These terminal conditions in turn are generated from contracts signed in the first J periods that do not convert to shipment until the following year.

xii) Let B_1 denote the total dollar budget constraint, for all J periods, related to recruiters; this includes their travel, office space, vehicles, etc. (For CY77 this was $71,357,325).

xiii) Let B_2 denote the total dollar budget constraint, for all J periods, related to advertising expense, including all national advertising, local advertising, RAD materials, direct mail costs, etc. (For CY77 this was about $16,070,000).

xiv) Let R denote the annual cost per recruiter including his salary, benefits, travel, office space, etc. (This value is assumed to be $21,190.)

xi) Let w_k denote the relative importance or penalty for not meeting a monthly shipping quota of the kth type ($k=1,2,\ldots,K$).

2.2 Definition of Decision Variables

i) Let $X_{i,j}$ denote the number of recruiters in region i, period j ($i=1,2,\ldots I$; $j=1,2,\ldots J$). (The current program has two main options regarding the assumed degree of mobility of recruiters:

a) it can be run in a manner so that only one level of recruiters is to be selected for each region for the J periods in the decision horizon; i.e. once the model selects this number of recruiters, it is committed to that level for the Region for the entire year. This mode of operation reflects the real-life fact that recruiters cannot be easily transferred from one region to another.

b) The other option is to remove this constraint so that the model is free to change the number of recruiters in each region from month to month. (The computer results given utilize the more conservative mode (a)).

ii) Let $Y_{i,j}$ denote the total advertising expenditures impacting in region i, period j ($i=1,2,\ldots,I$; $j=1,2,\ldots,J$) where total advertising includes RAD materials, direct mail, local advertising and all national media.

iii) It may also be noted that the variables discussed in (ix) of Section 2.1, i.e. the fractions of total contracts signed of the kth type in the jth period that convert to shipments v periods later, can either be treated as inputs or allowed to be decision variables also. The latter use would have the interpretation that the recruiter has a good deal of control in

how he utilizes the delays and hence can optimize the delays to best meet his schedule of quotas. The model then can be set up to choose the optimal set of $b_{v,j,k}$'s subject to the set of constraints:

$$\sum_{v=0}^{12} b_{v,j,k} = 1 - a_{j,k} \qquad (j=1,2,..,J\ ;k=1,2,...,K)$$

where $a_{j,k}$ is the inputted attrition factor (see factor (x) of 2.1) for the contracts of the kth type signed in the jth month and

$$b^U_{v,j,k} \leq b_{v,j,k} \leq b^L_{j,v,k}$$

where $b^L_{j,v,k}, b^U_{j,v,k}$ is some pre-set interval within which the recruiter can choose.

iv) Let $C_{i,j,k}$ denote the number of contracts of the kth type signed in the jth period from the ith region.

v) Let $S_{i,j,k}$ denote the number of shipments of the kth type of recruit in the jth period from the ith region.

2.3 Mathematical Model for the Budget Execution Model

2.3.1 (Objective Function)

There are several possible objectives for this mode which has been designed to minimize, within separate prescribed budget constraints for advertising and recruiters, weighted shortfalls from a given set of monthly or annual quotas for each category of recruits, utilizing a set of priorities for shortfalls for each class of recruits. Consider several of the formulations possible:

i) If it is desired to minimize the sum of shortfalls from given monthly quotas on shipments over the year, weighted by the type of shortfall, and if excesses in shipments in one month do not help offset deficits in another month, then the objective func-

tion would be:

$$\text{Minimize} \sum_{k=1}^{K} w_k \left\{ \sum_{j=1}^{\bar{J}} \left(Q_{j,k} - \sum_{i=1}^{I} S_{i,j,k}\right)^+ \right\} \tag{1}$$

where X^+ is X if X is greater than 0 and 0 elsewhere.

Hence this objective form calculates the sum of the national monthly shipment shortfalls (from the stated monthly national quotas $Q_{j,k}$) for each type of recruit (k), weights this summed shortfall by w_k (the relative priority for a shortfall of the kth type) and then sums this over the different types of recruits. Note that the use of the positive part (in contrast to the use of an absolute value) is to not explicitly penalize the allocations for any excess shipments in any given month; however, the model is indirectly penalized for any shipment excesses since resources expended in this manner do not add to meeting the monthly quotas, and hence constitute a waste of resources that are then not available for other time periods.

ii) Another formulation in a budget execution mode which may have some appeal is to simply minimize national shortfalls, over the total horizon, weighted by the relative penalties for national shortfalls for the different categories of recruits. This objective form then allows excesses in one month to compensate for deficits in other periods. This objective form is;

$$\text{Minimize} \sum w_k \left[\sum_{j=1}^{\bar{J}} Q_{j,k} - \sum_{j=1}^{\bar{J}} \sum_{i=1}^{I} S_{i,j,k} \right]^+ \tag{2}$$

so that the model is concerned only with the net shortfall over the total $\bar{J}$ periods. Note this model requires as input only total quotas for each of the different types of recruits, over the entire horizon. It then yields regional and monthly

targets for shipments which could be used in lieu of the present goaling model. If this model were to be used one should probably add as constraints upper bounds on the number of shipments that could be processed by the Recruit Training Centers in any given month.

iii) Finally consider one other variation, appealing in that it avoids the need for relative priority factors, namely the w_k's and closely approximates the system actually in use. The approach would be to specify explicit quotas, either monthly or for the entire horizon, for only one type of recruit , e.g. "totals", and then to impose as a side constraint that at least some prespecified percent of the total shipments be high school graduates; this side constraint again could be on a monthly basis or for the horizon as a whole. In this case the objective form is either (1) or (2), with K=1, and w_1=1, and with an additional side constraint of the type

$$\sum_{j=1}^{\bar{J}} \sum_{i=1}^{I} S_{i,j,2} \geq \mathrm{PERC} \left(\sum_{j=1}^{\bar{J}} \sum_{i=1}^{I} S_{i,j,1} \right)$$

where PERC is the minimum percent of total shipments that should be high school graduates.

2.3.2 Constraints in the Budget Execution Model

The formal constraints in the model are:

$$C_{i,j,k} = P_{i,j,k} \prod_{t=0}^{L_k} X_{i,j-t}^{e_{t,k}} \cdot Y_{i,j-t}^{f_{t,k}} \qquad \begin{pmatrix} i=1,2,\ldots,I; \\ j=1,2,\ldots,J; \\ k=1,2,\ldots,K \end{pmatrix} \tag{3}$$

where the above is the production function relating to contracts and $e_{t,k}$, $f_{t,k}$ are the elasticities discussed in the input section, related to recruiters and advertising respectively.

The next set of constraints relate to the budgets on recruiters and advertising:

$$\sum_{j=1}^{J} R \left(\sum_{i=1}^{J} X_{i,j} \right) \leq B_1 \tag{4}$$

$$\sum_{j=1}^{J} \sum_{i=1}^{I} Y_{i,j} \leq B_2 \tag{5}$$

Since the objective function is based on quota shortfalls relative to shipments, and the allocation of recruiters and advertising is thru contracts, it is necessary to convert contracts to shipments. This is done through the delay factors of the DEP program. If the delay factors are treated as decision variables, then one has additional constraints:

$$\sum_{v=0}^{12} b_{v,j,k} = 1-a_{j,k} \quad (j=1,2,\ldots.,12; \; k=1,2,\ldots..K) \tag{6}$$

$$b^{L}_{v,j,k} \leq b_{v,j,k} \leq b^{u}_{v,j,k} \quad (v=0,1,2,..J; \; j=1,2,\ldots J; \; k=1,2,\ldots K) \tag{7}$$

Otherwise the $b_{v,j,k}$'s are simply inputs. The shipments then, as a function of the contracts, is given as

$$S_{i,j,k} = \sum_{v=0}^{12} C_{i,j-v,k} \cdot b_{v,j-v,k} \tag{8}$$

i.e., the total shipments in period j, region i of type k, is simply the sum of the appropriate number of contracts signed v periods earlier (v=0,1,2.....,12) times the fraction of those that ship v periods after they sign.

Finally if one wishes the model to select the best single value (for each region) for the number of recruiters for the J periods

in the decision horizon, then in order to freeze the number of recruiters at that level, one must also add

$$X_{i,1} = X_{i,j} \quad (i=1,2....I;\ j=2,3......J) \tag{9}$$

If one wishes to allow the number of recruiters to vary within a given region over the horizon, then one need add no additional constraints of type (9).

2.4 Mathematical Model for the Budget Generation Model

2.4.1 Objective Function

This model is designed to determine the minimum total budget, as well as the optimal split of dollars between recruiting and advertising, needed to meet a given set of quotas. Hence given the same notation as earlier, the objective function is to minimize the sum of the left hand side of constraints (4) and (5) of Section 2.3.2.

2.4.2 Constraints for the Budget Generation Model

The constraints are to meet a given set of national quotas, either on a monthly or total horizon basis, i.e.

$$\sum_{i=1}^{I} S_{i,j,k} \geq Q_{j,k} \quad (j=1,2......\bar{J};\ k=1,2.........K)$$

if monthly quotas are given, or

$$\sum_{j=1}^{\bar{J}} \sum_{i=1}^{I} S_{i,j,k} \geq \sum_{j=1}^{\bar{J}} Q_{j,k} \quad (k=1,2......K)$$

if only horizon quotas are given.

The other constraints (3), (6), (7), (8), and (9) of Section 2.3.2 apply as before.

3.0 THE ENLISTMENT PREDICTION MODELS AND THEIR VALIDATION

3.1 Considerations in Selection of Production Model

Econometric methods were used to ascertain the relationship between the number of signed contracts and the various factors which influence the propensity to enlist in the Navy. These factors include the amount of money spent on advertising and promotion, the number of recruiters, the number of leads, the overall number of high school graduates, etc. Separate relationships were developed for total contracts, and for contracts signed by high school graduates. In addition, the number of leads was related econometrically to a similar set of variables. Hence, we have a three-equation system: an equation with leads as the dependent variable, an equation with total enlistments as the dependent variable and leads as one of the explanatory variables, and a similar equation for enlistments by high school graduates.

Numerous conditions make such model-building a complicated and difficult task. These conditions and methods of solution are discussed below.

3.1.1 The Short Time Series Available

The levels of advertising expenditures have been measured for only two years 1976 and 1977, resulting in only 24 monthly observations. Such a short time series make it difficult to estimate a model with more than one or two explanatory variables. This problem was solved by disaggregating the data into districts and using a pooling technique to estimate the model. This disaggregation resulted in 1032 observations (24 months times 43 districts). Hence, both cross sectional as well as a time series analysis is performed. The technique of "Dummy Variable Regression" was used to pool the data by creating an indicator variable (or dummy variable) for each of the 43 districts (actually only 42 dummy variables were used). This method allows each district to have its own "constant term" in the model which represents the propensity to enlist which is unique to that district. An indicator variable is either zero or one. It has a value of one if the observation is from

the district, and a value of zero if the observation is from some other district. It is also of interest to note that the dollar expenditures were all adjusted for inflation.

3.1.2 Seasonal Variation

There are seasonal variations in enlistments and in leads. These variations were ascertained in the models by assigning an indicator variable to each month. Hence, we permit each month to have a unique level of enlistments and leads.

3.1.3 Extraordinary Events

Two special events impacted both enlistments and leads: television advertising was extensively used in 1977 but not in 1976; and the GI-bill expired at the end of 1976. These events were handled by using indicator variables, one for each event.

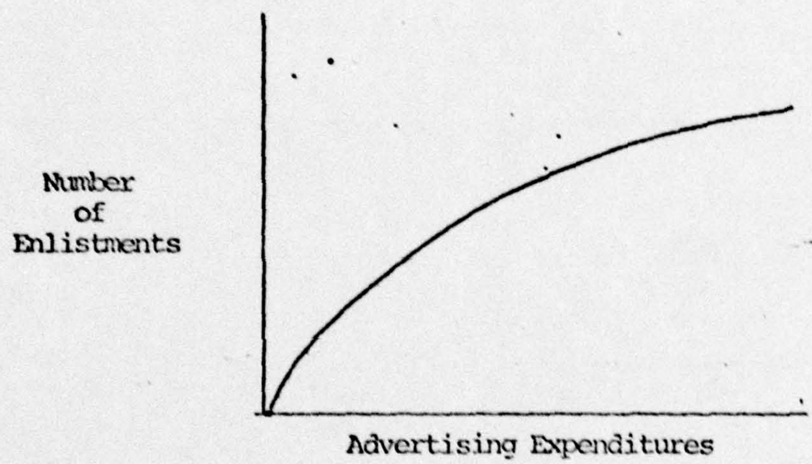
3.1.4 District Size Variations

It is generally true that larger districts (in terms of population and QMA) produce more enlistments than smaller districts. Because of this size variation, it has been the practice to allocate more advertising and recruiters to the larger districts. This resource allocation procedure has produced a strong relationship between advertising and enlistments (and also recruiters and enlistments). Terming this tendency of the Navy to allocate resources in proportion to the number of enlistments as the "allocating relationship", it is important to distinguish this from the "market relationship," which is the response of the market to recruiting and advertising efforts. The "allocation relationship" says that when enlistments are high, resources will be high; "the market relationship" says that when resources are high, enlistments are high. An analysis which used regression analysis to estimate a model with enlistments as the dependent variable would confuse the two relationships. If one naively performed such a regression to estimate the market relationship, the results would be misleading because the model would overstate the impact of expenditures on enlistments. This problem was attenuated by removing the allo-

cation relationship from the data by dividing all of the variables by the level of labor force in each district. Hence, the dependent variable became enlistment per capita (or leads per capita).

3.1.5 Non-linear Relationships

A linear regression model with per capita enlistments as the dependent variable and per capita advertising and per capita recruiters as explanatory variables would assume a linear relationsihp between enlistments and recruiting effort. In such a relationship, the addition of one dollar of advertising would have the same absolute impact on enlistment for all base levels of advertising. Such a constant impact is generally thought to be too optimistic because of the notion of diminshing returns. As additional dollars are spent on advertising, the impact of each addition becomes less. Such a relationship is shown below.



This non-linear relationship was captured by using logrithms of all variables.

3.1.6 Carry-over Relationships

Recruiting efforts have an impact long after they take place, i.e., there is a carry-over from one period to the next. Such a carry-over can be captured by including "lagged values" of the explanatory variables in the models.

That is, enlistments would be regressed on current levels of advertising <u>and</u> historical levels. However, the inclusion of historical levels causes a multitude of problems, such as correlation between the current level and historical levels.

One solution to such problems and the one utilized in this analysis, is to place some type of constraint on the solution. The procedure is to assume that the largest impact of a factor such as advertising is in the current or immediately preceding month and that subsequent impacts decline geometrically over time. Hence, advertising's impact is allowed to peak in either the current or immediately preceding period. Such an assumption produces a so-called Koyck distributed lag model which has the lagged value of the dependent variable as an explanatory variable.

The exact procedure can be seen in the following simple model. Let E_t denote enlistments in month t, and let A_t denote the level of advertising expenditures in month t. The carry-over relationship is given by

(1) $E_t = a + b_o A_t + b_1 A_{t-1} + b_2 A_{t-2} + \ldots\ldots\ldots + e_t$ where e_t is a random disturbance. The Koyck assumption of geometrically declining advertising impact can be expressed as the following;

(2) $$E_t = a_o \quad b_o A_t + b_1 A_{t-1} + \beta(1-\lambda) \sum_{i=0}^{\infty} \lambda^i A_{t-i-2}$$

where $o < \lambda < 1$ is the decay factor. Using this assumption, we can reduce the carry-over model to

(3) $$E_t = a(1-\lambda) + b_o A_t + (b_1 - \lambda b_o) A_{t-1} + \lambda E_{t-1} + U_t$$

Hence we have a model with current advertising (A_t), and current advertising lagged one period (A_{t-1}), and lagged enlistments (E_{t-1}) which can be estimated. Once estimates are obtained for the coefficients of Equation (3), the relationships in (2) can be used to obtain estimates of the coefficients in Equation (1), and the short term and long run impacts of advertising identified.

3.2 Regression Results

The regression equation for total enlistments in district i, period t is given by:

$$\log\left\{\frac{E_{it}}{LF_{it}}\right\} = A_o + A_1 \cdot D_1 + \ldots + A_{42} \cdot D_{42} + b_1 \cdot M + \ldots b_{11} \cdot M_{11}$$

$$+ C_1 \cdot Y + C_2 \cdot G + d_1 \cdot \log\left\{\frac{A_{it}}{LF_{it}}\right\} + d_2 \cdot \log\left\{\frac{R_{it}}{LF_{it}}\right\}$$

$$+ d_3 \cdot \log\left\{\frac{L_{i,t}}{LF_{i,t}}\right\} + d_4 \cdot \log\left\{\frac{E_{i,t-1}}{LF_{i,t-1}}\right\}$$

$$+ d_5 \cdot \log\left\{\frac{T_{it}}{LF_{it}}\right\}$$

E	=	Total enlistments
LF	=	Labor force
i	=	district
t	=	time period
D	=	District indicator variable
M	=	Monthly indicator variable
Y	=	Year indicator variable
G	=	GI bill expiration indicator variable
A	=	Total advertising and promotion expenditure
R	=	Number of recruiters
L	=	Number of leads
T	=	Total high school graduates

The estimates of the slope coefficients are given below, along with their standard errors and t-statistics.

Total Enlistments Model

Variable	Coefficient Estimate	Standard Error	t-value
$\log \left\{\frac{A_{it}}{LF_{it}}\right\}$	.062	.022	2.82
$\log \left\{\frac{R_{it}}{LF_{it}}\right\}$	.291	.084	3.46
$\log \left\{\frac{L_{it}}{LF_{it}}\right\}$	.039	.013	3.00
$\log \left\{\frac{T_{it}}{LF_{it}}\right\}$	.458	.100	4.58
$\log \left\{\frac{E_{i,t-1}}{LF_{i,t-1}}\right\}$	.364	.026	14.0

All of these coefficient estimates are statistically significant at the 0.01 level.

A similar regression was performed with enlistments by high school graduates as the dependent variable (actually the logarithm of high school enlistment divided by the labor force). The results are given below.

High School Graduates Enlistments Model

Variable	Coefficient Estimates	Standard Error	t-value
$\log \left\{ \frac{A_{it}}{LF_{it}} \right\}$	.073	.020	3.65
$\log \left\{ \frac{R_{it}}{LF_{it}} \right\}$	.391	.095	4.12
$\log \left\{ \frac{T_{it}}{LF_{it}} \right\}$	.356	.113	3.15
$\log \left\{ \frac{L_{it}}{LF_{it}} \right\}$	.051	.015	3.40
$\log \left\{ \frac{E_{it-1}}{LF_{it-1}} \right\}$	.353	.028	12.61

Again, all estimates are significant at the 0.01 level.

The model for leads included all the dummy variables, and variables for total advertising, and high school graduates. The results are given below:

Leads Model

Variable	Coefficient Estimates	Standard Error	t-value
$\log \left\{ \frac{A_{it}}{LF_{it}} \right\}$	.160	.041	3.90
$\log \left\{ \frac{T_{it}}{LF_{it}} \right\}$	.611	.137	4.46

Both estimates are significant at the 0.01 level.

After developing these respective elasticities from the district level observations (of which there were 43) the coefficients for the regional dummy variables, numbering 6, were obtained by rerunning the log linear regressions at the regional level but with the elasticities obtained at the district level. In this way the final predictive models, after aggregating the regional/monthly specific terms into one constant (namely the $P_{i,j,1}$'s) become:

$$\begin{array}{c}\text{\# of total contracts}\\ \text{(in thousands)}\\ \text{in period j}\\ \text{from region i}\end{array} = P_{i,j,1} \cdot \left(\begin{array}{c}\text{no. of recruiters}\\ \text{(in thousands)}\\ \text{in period j,}\\ \text{region i}\end{array}\right)^{.29} \times \left(\begin{array}{c}\text{no. of recruiters}\\ \text{(in thousands)}\\ \text{in period j-1,}\\ \text{region i}\end{array}\right)^{.106} \times \left(\begin{array}{c}\text{no. of recruiters}\\ \text{(in thousands)}\\ \text{in period j-2,}\\ \text{region i}\end{array}\right)^{.039} \times$$

$$\left(\begin{array}{c}\text{Millions of dollars}\\ \text{of total advertising}\\ \text{in period j, region i}\end{array}\right)^{.125} \times \left(\begin{array}{c}\text{Millions of dollars}\\ \text{of total advertising}\\ \text{in period j-1,}\\ \text{region i}\end{array}\right)^{.045} \times \left(\begin{array}{c}\text{Millions of dollars}\\ \text{of total advertising}\\ \text{in period j-2,}\\ \text{region i}\end{array}\right)^{.017}$$

The 12x6 matrix of $P_{i,j,1}$'s (which are the result of running the regression with regional monthly data using the elasticities developed earlier) is presented in Appendix A.

The similar equation for number of HSG contracts is:

$$\left\{\begin{array}{c}\#\text{ of HSG contracts}\\ \text{(in thousands) from}\\ \text{region } i\text{, period } j\end{array}\right\} = P_{i,j,2} \left\{\begin{array}{c}\text{number of recruiters}\\ \text{(in thousands) in}\\ \text{period } j\text{, region } i\end{array}\right\}^{.391} \times \left\{\begin{array}{c}\text{number of recruiters}\\ \text{(in thousands) in}\\ \text{period } j-1\text{, region } i\end{array}\right\}^{.138} \times \left\{\begin{array}{c}\text{number of recruiters}\\ \text{(in thousands) in}\\ \text{period } j-2\text{, region } i\end{array}\right\}^{.049}$$

$$\left\{\begin{array}{c}\text{Millions of dollars}\\ \text{of total advertising}\\ \text{in period } j\text{, region } i\end{array}\right\}^{.081} \times \left\{\begin{array}{c}\text{Millions of dollars}\\ \text{of total advertising}\\ \text{in period } j-1\text{, region } i\end{array}\right\}^{.029} \times \left\{\begin{array}{c}\text{Millions of dollars}\\ \text{of total advertising}\\ \text{in period } j-2\text{, region } i\end{array}\right\}^{.01}$$

The 12x6 matrix of $P_{i,j,2}$'s are also in Appendix A. The following summary Table shows the relative impact of advertising and recruiter on HSG contracts and non-HSG contracts.

CONTRACT ELASTICITIES

	Total Contracts	HGS Contracts	Non HSG Contracts
Advertising			
Current period	.125	.081	.210
One period earlier	.045	.029	.079
Two periods earlier	.017	.01	.031
Number of Recruiters			
Current period	.291	.391	.085
One period earlier	.106	.138	.042
Two periods earlier	.039	.047	.024

As an example a one percent increase in the amount of advertising in the current period would bring about on the average, if everything else remained constant, a .081% increase in the number of HSG contracts signed in the current period. Note that advertising has a much higher relative impact on non-high school contracts than on HSG's ; also recruiters have a much larger relative impact on HSG contracts than for the non HSG segment.

3.3 "Validation" of Predictive Models

In order to assess how well the predictive models perform plots of the comparisons of the actual numbers of contracts versus the predicted, using the actual budget expenditures as experienced in CY77, were made. Plots of the comparisons by month over the entire country, as well as for, each of the six regions, are included for both HSG contracts and for all contracts. Figures 1 and 2 are for the country as a whole, respectively, for total contracts and HSG contracts; Figures 3-14 are the regional comparisons; 8-18 being for HSG contract and 9-14 being for total contracts.

The straight line shown on the plots would be the pattern if the prediction were perfect, i.e., if actual contracts exactly equal the predicted number of contracts. As one can see the points are generally quite close to the line, with the exception sometimes of one point (that point in the upper right hand portion of the plot) which represents the results for December 1976. This was the last month for recruits to receive the GI bill and hence, even though a dummy variable was used to try to deal with this anomaly there is some inaccuracy for this one period.

FIGURE 1

U.S. TOTAL
ACTUAL VS. PREDICTED
TOTAL CONTRACTS
(IN 1000)

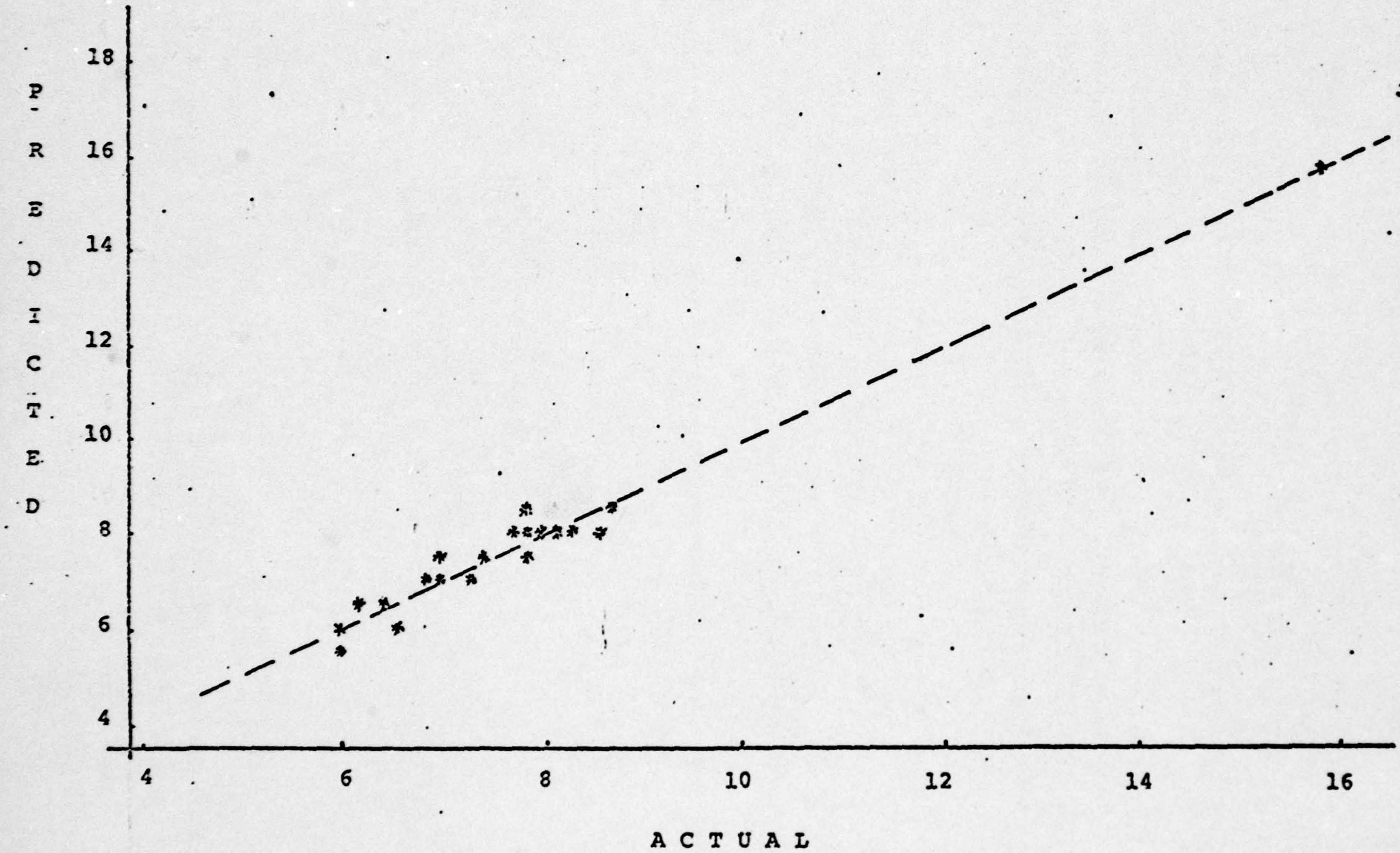


FIGURE 2

U.S. TOTAL
ACTUAL VS. PREDICTED
HSG. CONTRACTS
(IN 1000)

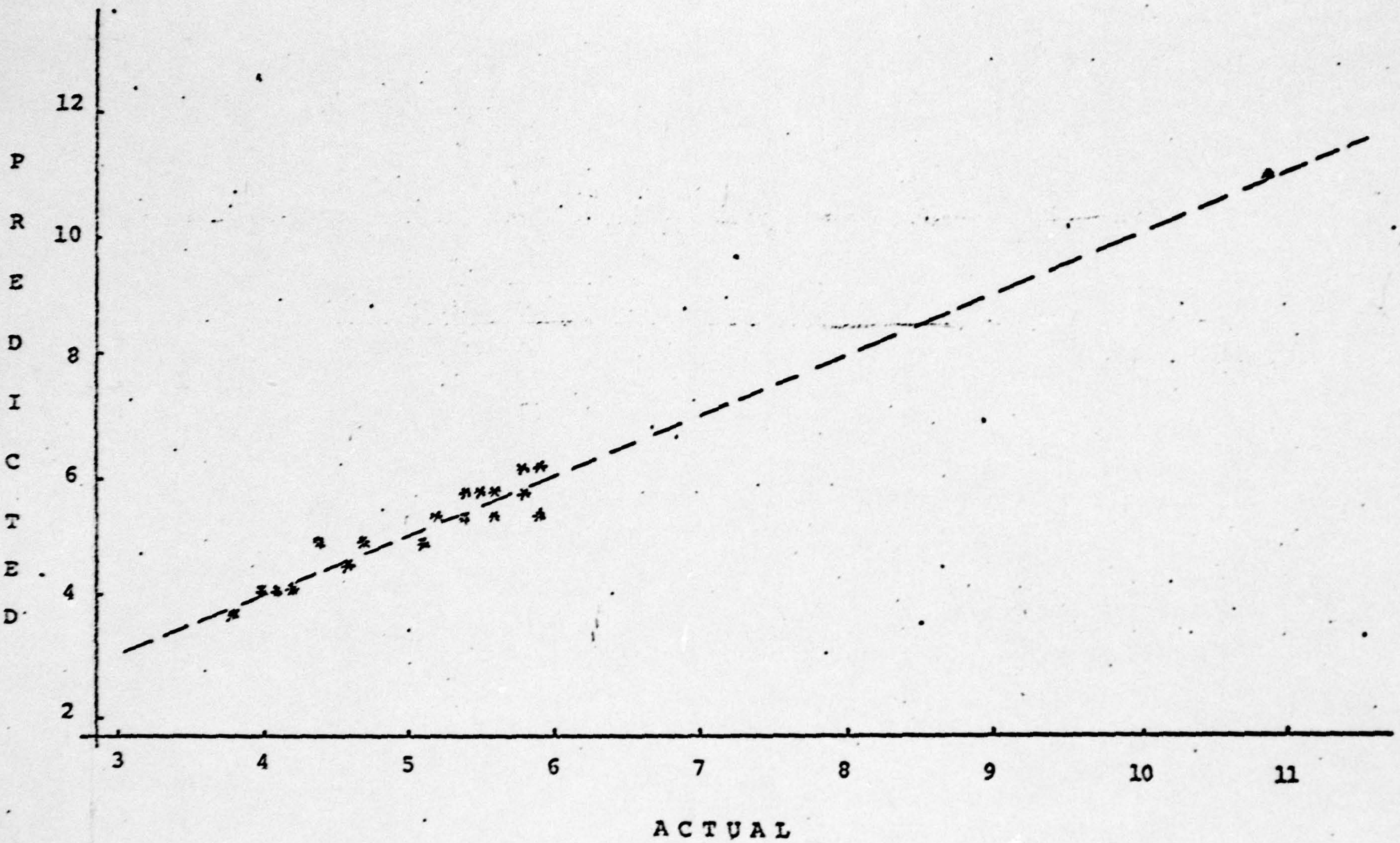


FIGURE 3

PLOT OF THE ACTUAL VS PREDICTED CONTRACTS (HSG)
NBA=100

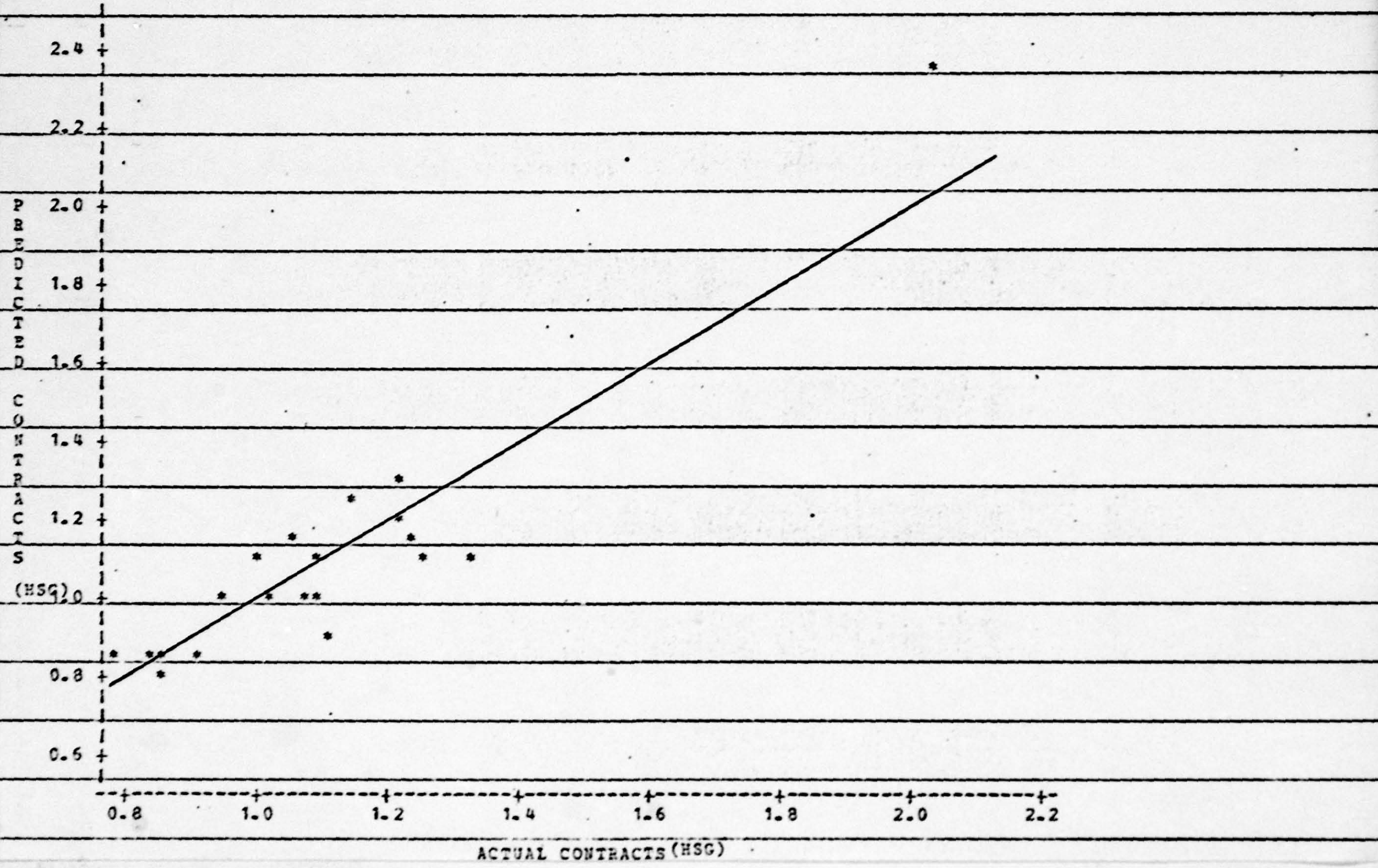


FIGURE 4

PLOT OF THE ACTUAL VS PREDICTED CONTRACTS (HSG)
NEA=300

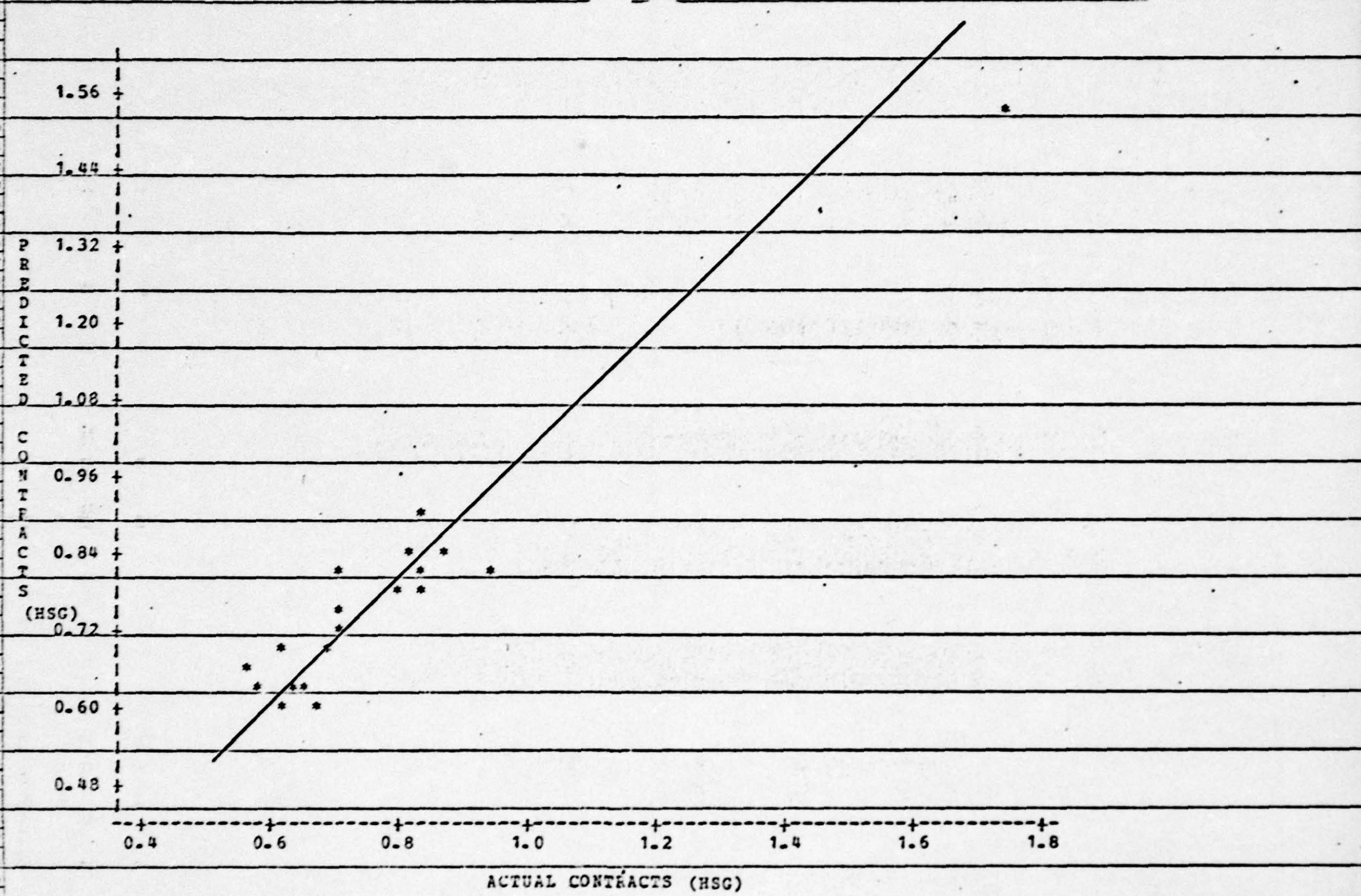


FIGURE 5

PLOT OF THE ACTUAL VS PREDICTED CONTRACTS (HSG)
NRA=400

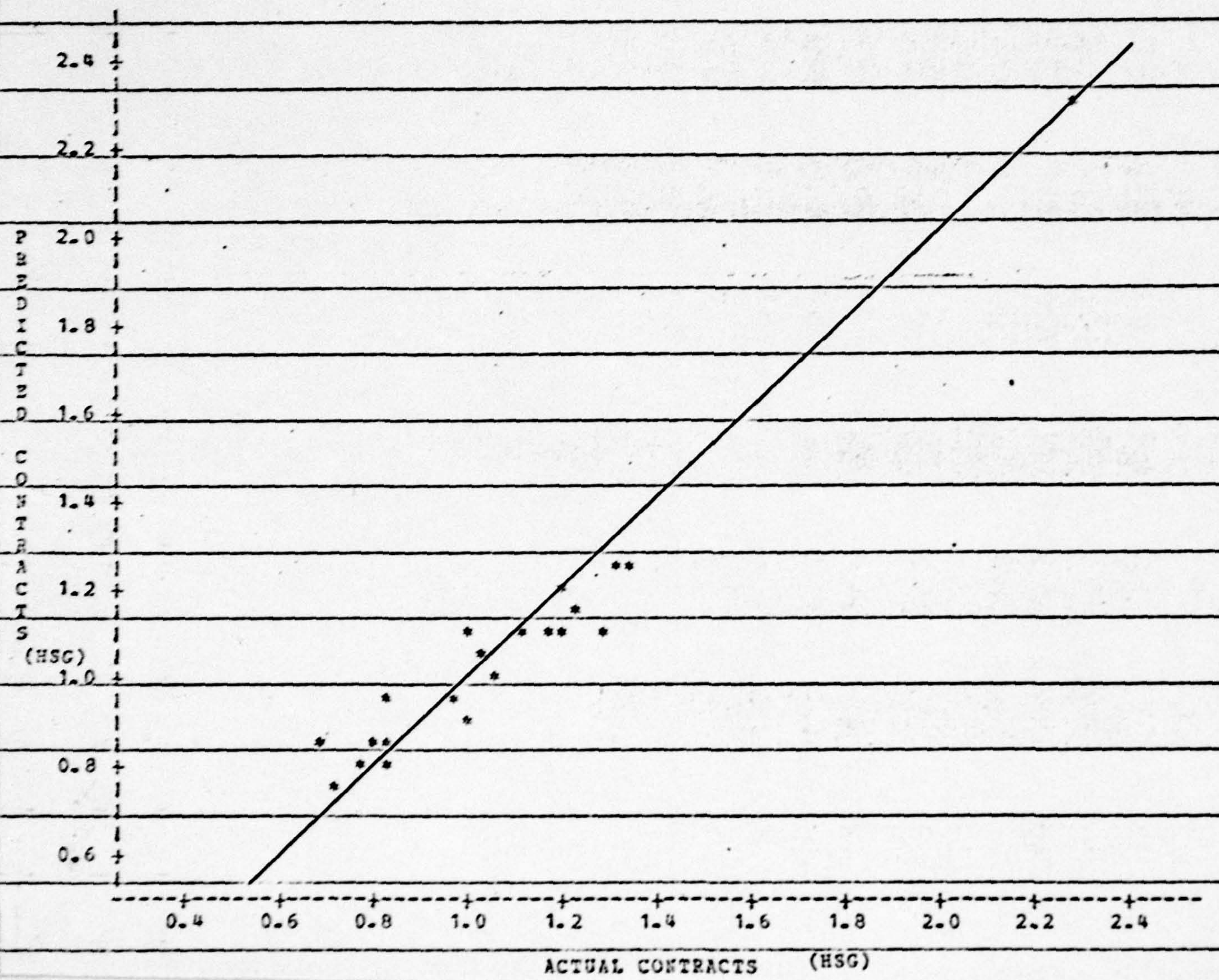
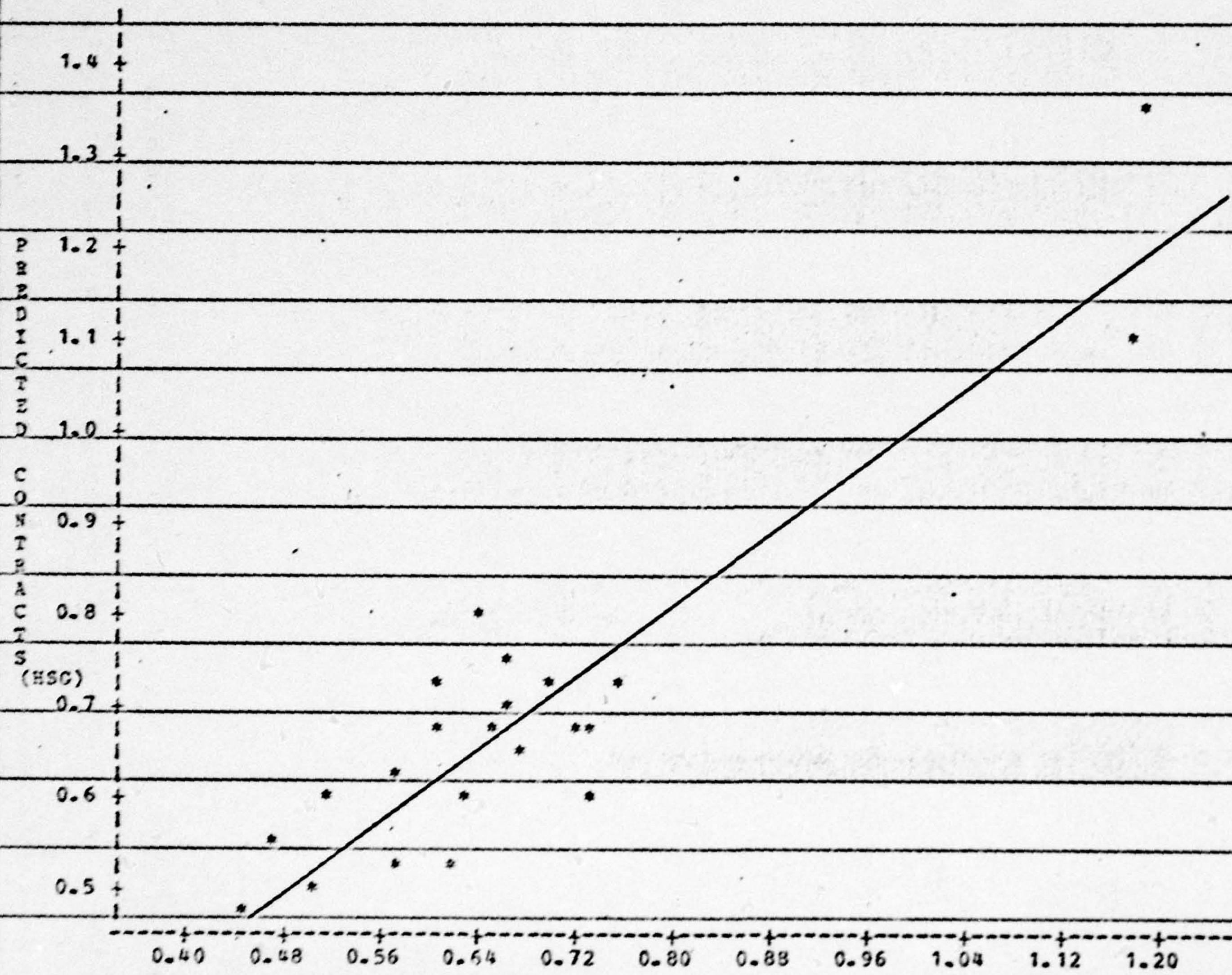


FIGURE 6

PLOT OF THE ACTUAL VS PREDICTED CONTRACTS (HSC)

NRA=500



ACTUAL CONTRACTS (HSC)

FIGURE 7

PLOT OF THE ACTUAL VS PREDICTED CONTRACTS (HSG)
NRA=700

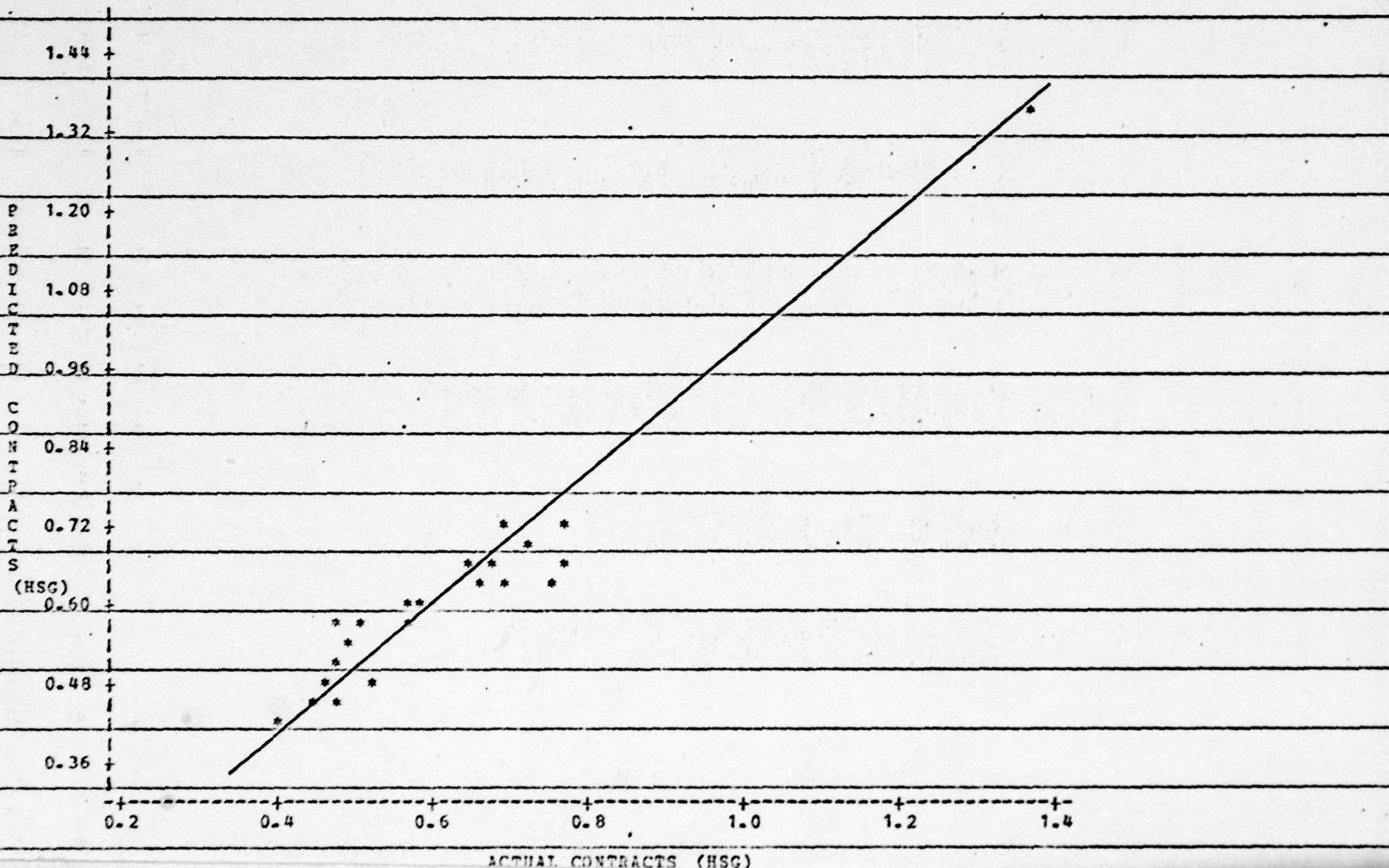


FIGURE 8

PLOT OF THE ACTUAL VS PREDICTED CONTRACTS (HSG)
NRA=800

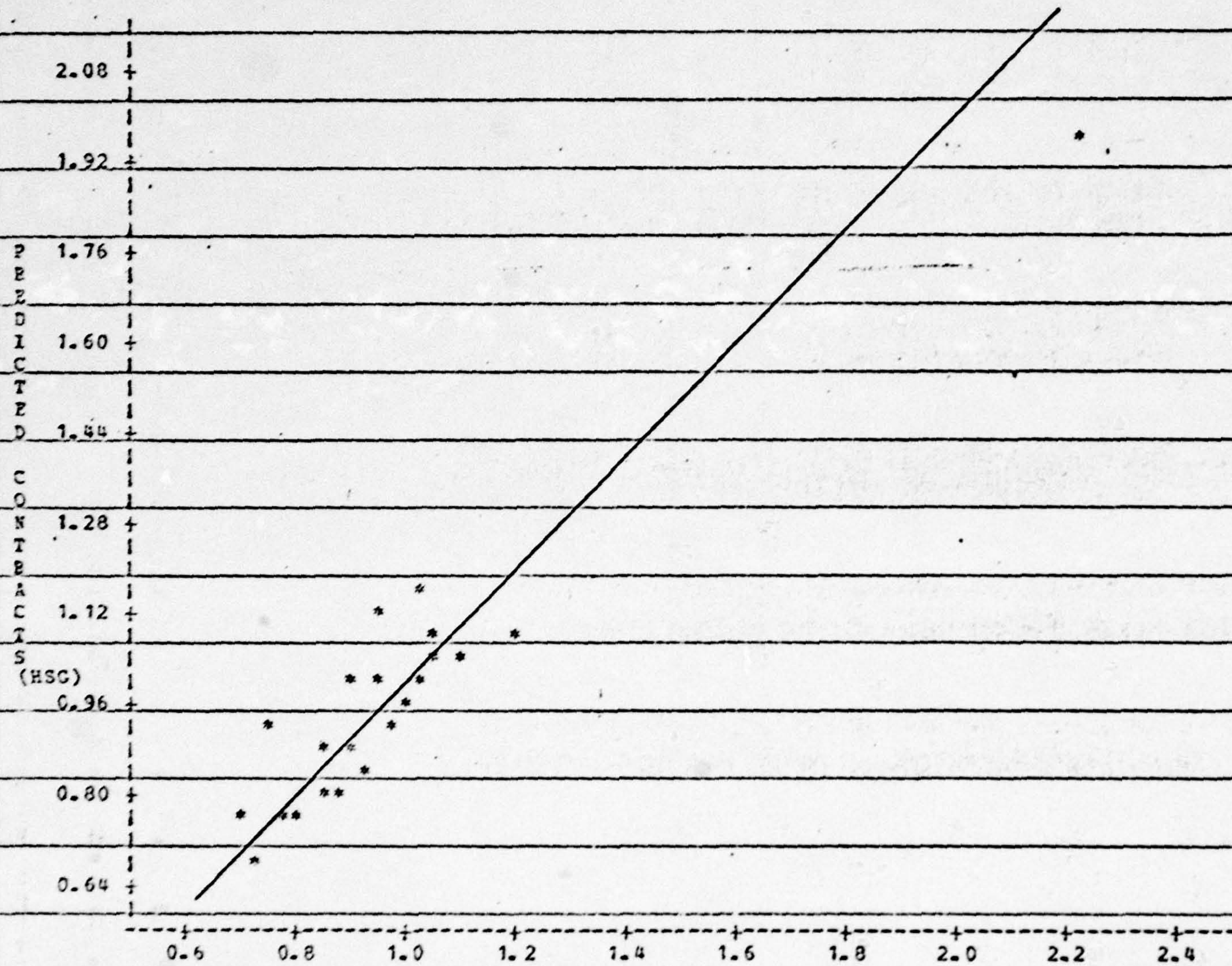


FIGURE 9

PLOT OF ACTUAL VS PREDICTED CONTRACTS
FOR TOTAL MALE CONTRACTS
NRA=100

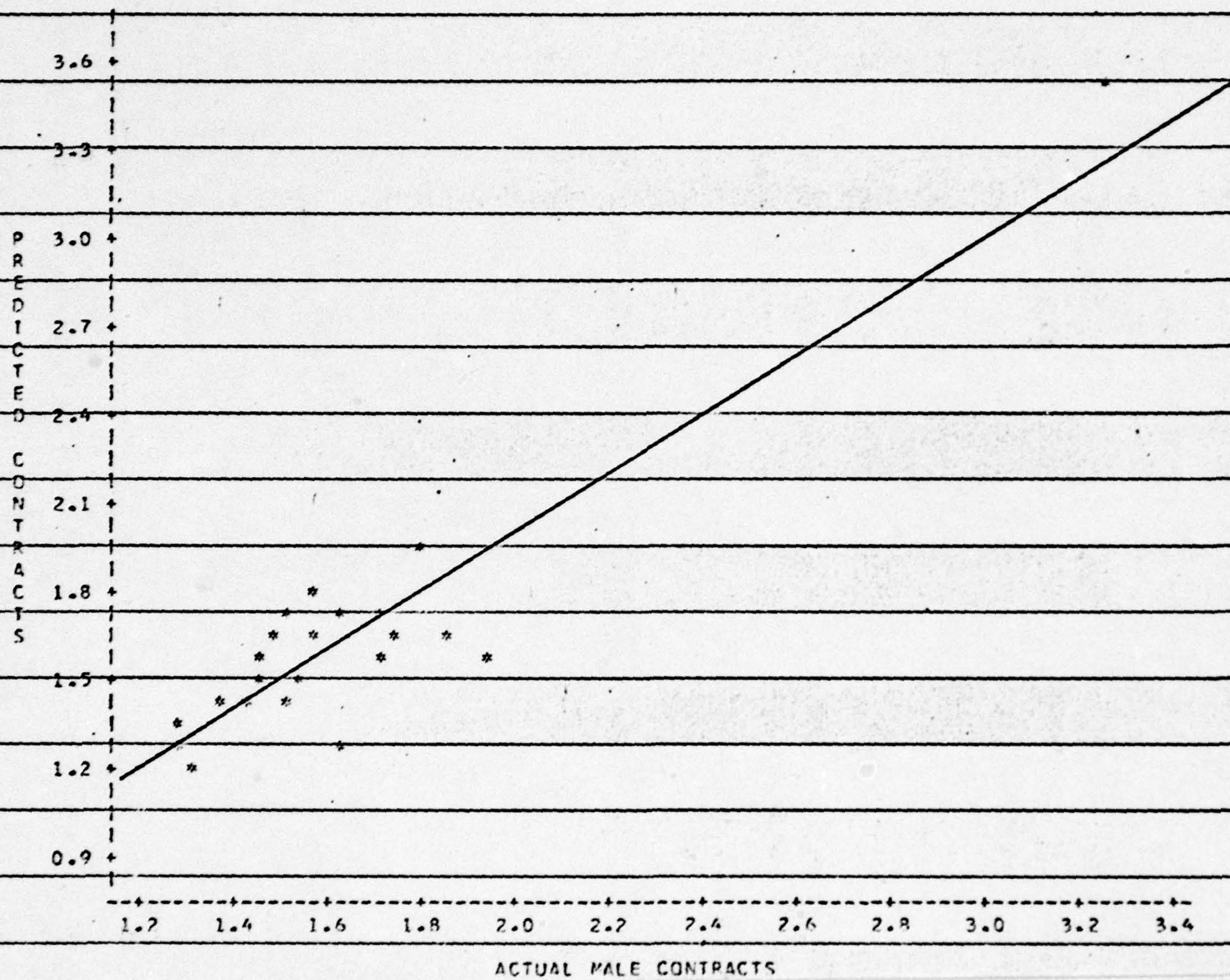


FIGURE 10

PLOT OF ACTUAL VS PREDICTED CONTRACTS
FOR TOTAL MALE CONTRACTS
NRA=300

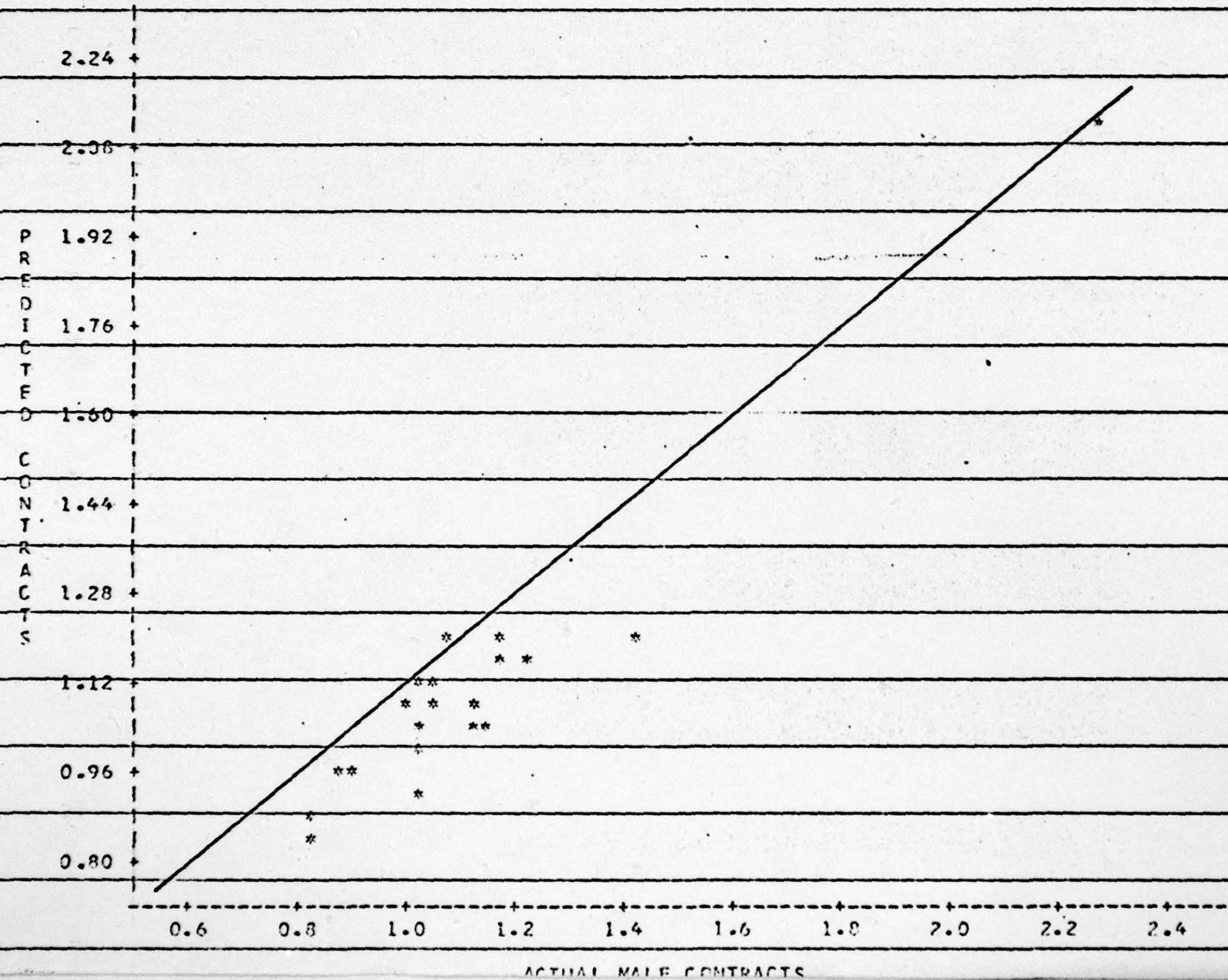


FIGURE 11

PLOT OF ACTUAL VS PREDICTED CONTRACTS
FOR TOTAL MALE CONTRACTS
NRA=400

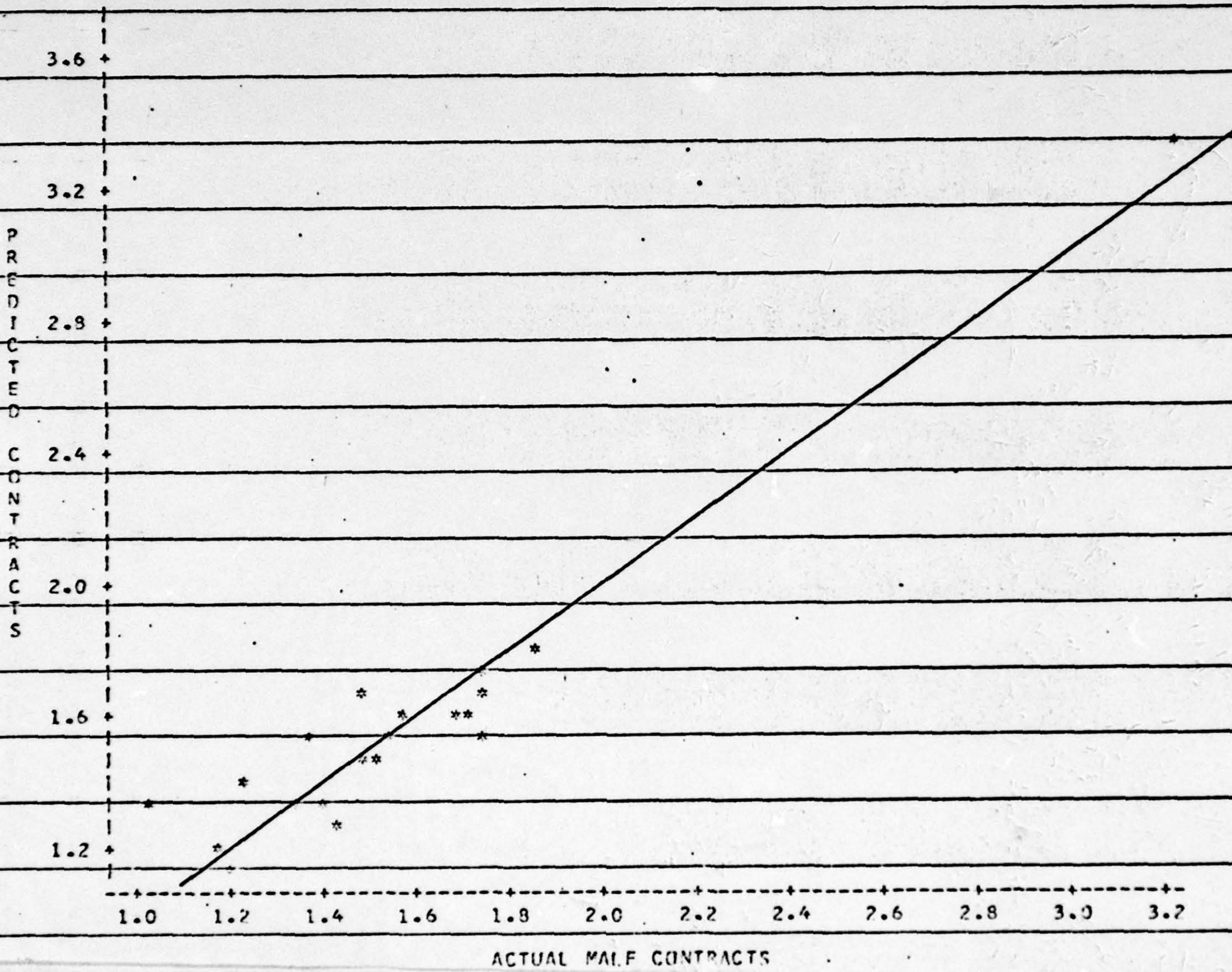


FIGURE 12
PLOT OF ACTUAL VS PREDICTED CONTRACTS
FOR TOTAL MALE CONTRACTS
NRA=503

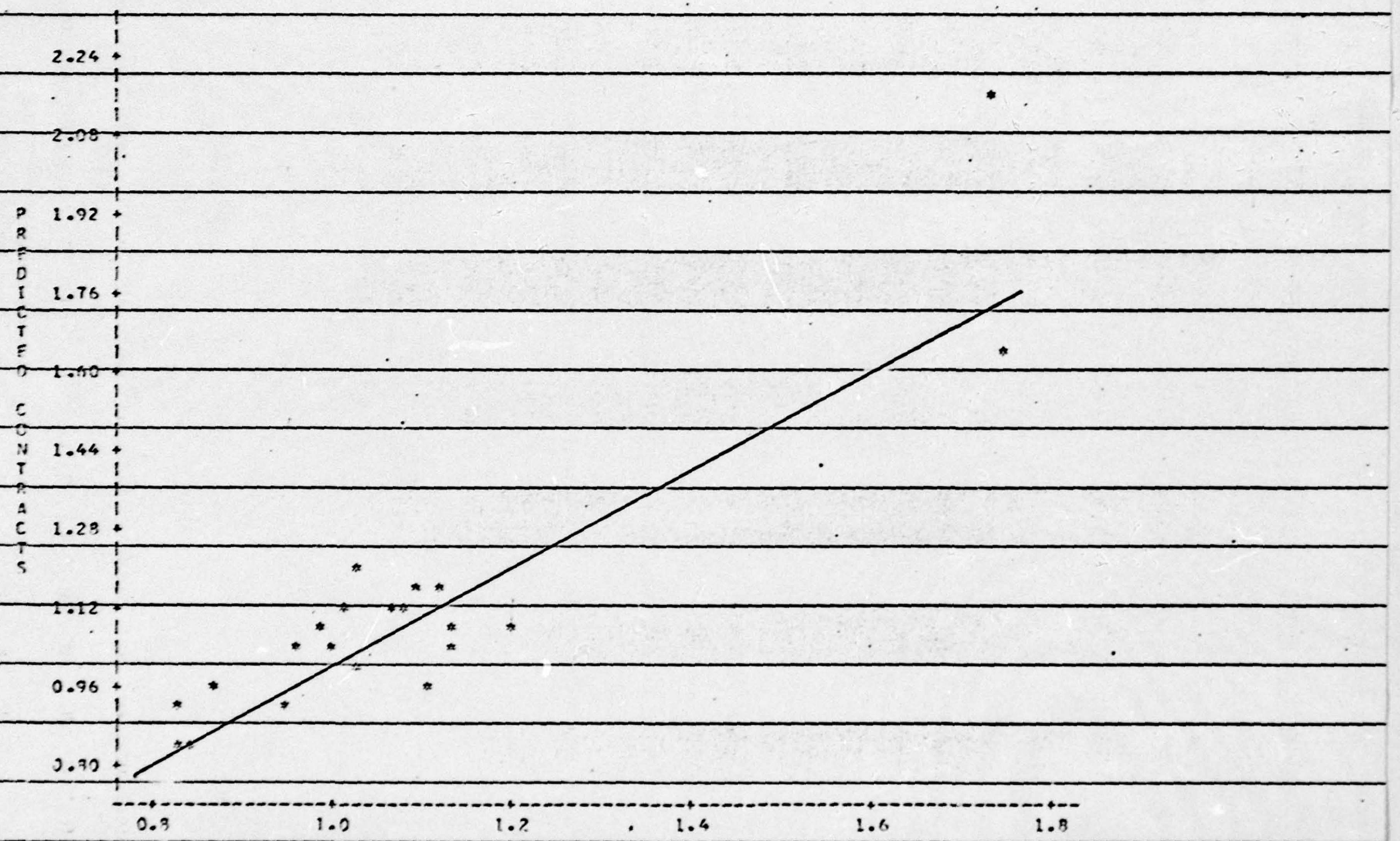


FIGURE 13

PLOT OF ACTUAL VS PREDICTED CONTRACTS
FOR TOTAL MALE CONTRACTS
NRA=700

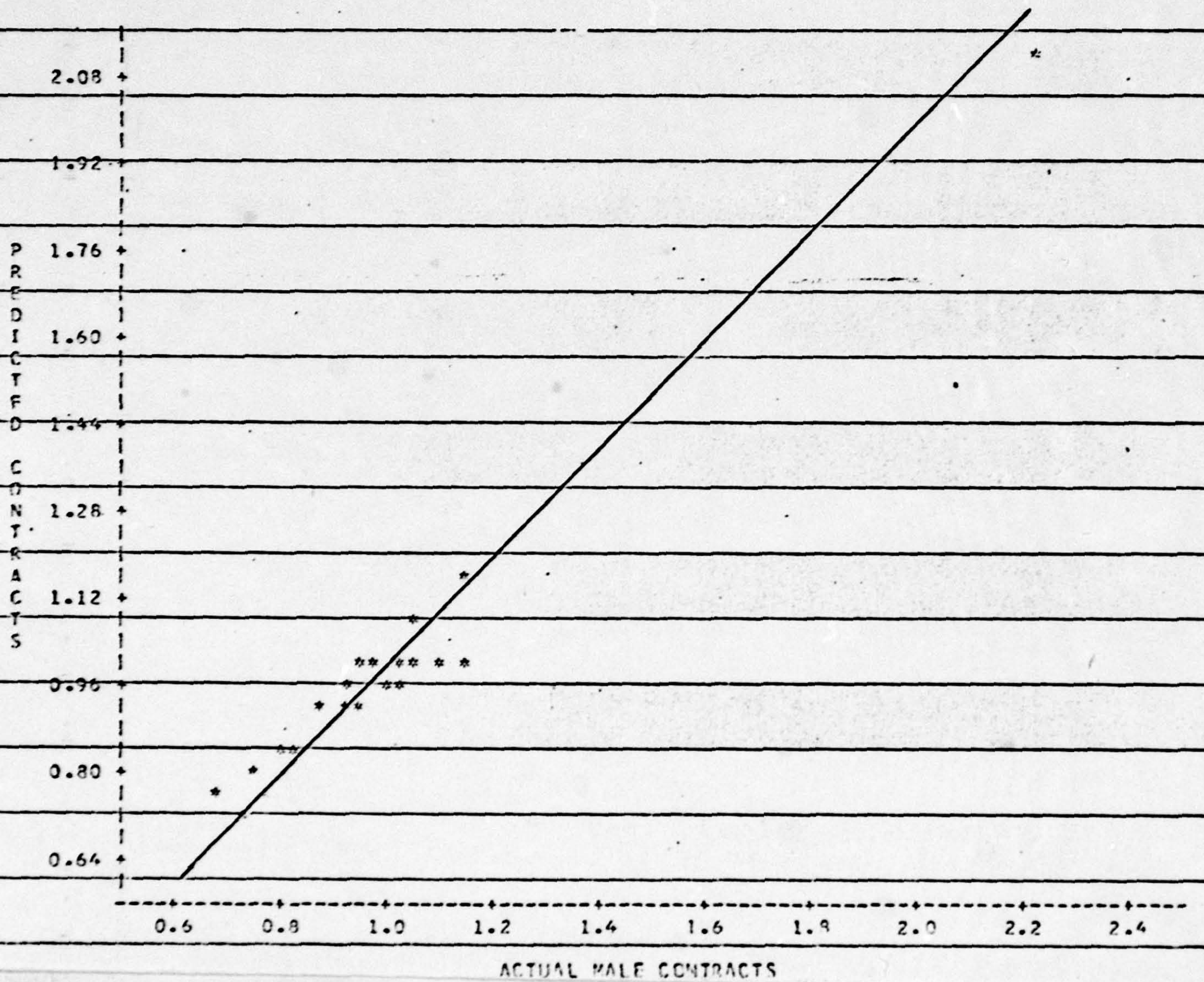
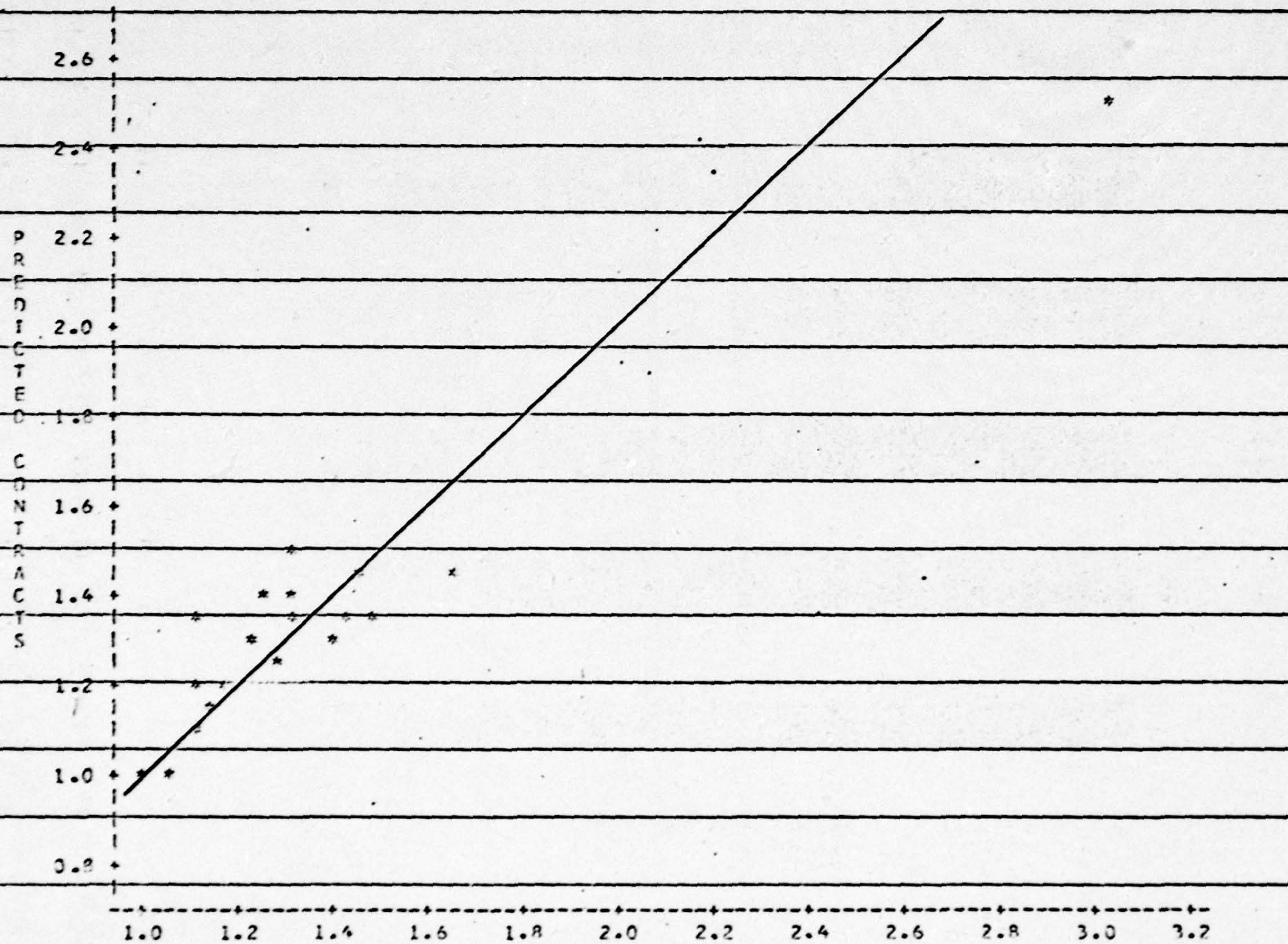


FIGURE 14

~~PLOT OF ACTUAL VS PREDICTED CONTRACTS~~
FOR TOTAL MALE CONTRACTS
NRA=800



4.0 "VALIDATION" OF DESCRIPTIVE ABILITY OF ALLOCATION MODEL

Moving from the response functions to the important budget allocation model, it was decided for validation purposes to exercise the model for a representative recent year for which complete data was available. The validation strategy had two parts: 1) first to exercise the model with the same budget expenditures as actually occurred to see if it predicts the shipments that actually occurred reasonably well. This is a test of the general descriptive ability of the model. 2) Secondly, using the same initial conditions that were faced by the actual decision-makers in that year, compare the decisions and recommendations coming out of the model with the decisions and results actually occurring. It was felt that only by comparing the optimization results could one also establish a credible degree of reasonableness in the final allocation program.

The year chosen was calendar year 1977, chosen since it is the latest year for which detailed advertising data by district was available. In addition since the program must take into account certain "initial" conditions from the previous year and certain terminal conditions for the year following the decision year, it became clear that CY77 was the appropriate year to test the model.[1]

First consider the descriptive ability of the allocation model. First one inputs to the allocation model: the actual monthly advertising and recruiter allocation made in CY77, the "stock" of advertising and numbers of recruiters in place in 1976, the number of men in the delayed entry program in 1976; then one predicts the following over the country (see Table 1).

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[1] It may be noted that the plan is to exercise the model for CY78 when the detailed advertising for that period becomes available.

TABLE 1: COMPARISON OF ACTUAL VERSUS PREDICTED CONTRACTS AND SHIPMENTS FOR ACTUAL BUDGET ALLOCATIONS

	Total Contracts	Predicted Total Contract	Actual High School Graduate Contract	Predicted Actual HS Grad. Contract	Actual Total Shipments	Predicted Total Shipments
Jan. 77	7029	7022	4748	4745	8529	8605
Feb. 77	7239	7198	4729	4697	6833	6590
Mar. 77	7798	7849	5058	4944	5971	6295
Apr. 77	6397	6713	4139	4173	5051	5347
May 77	6195	6621	4028	4228	5802	6125
June 77	8251	8505	5579	5806	10203	11,611
July 77	7812	7720	5194	5193	10667	11,215
Aug. 77	8617	7956	5615	5347	11542	12,194
Sept.77	7498	7690	4422	4796	11189	10,467
Oct. 77	6013	5990	3784	3745	6815	7,367
Nov. 77	6849	6838	4059	4046	5830	6291
Dec. 77	6926	6909	3972	4021	4206	3859
TOTAL for Yr.	86624	87011	55327	55741	92638	95966

Note that for the yearly totals, the model slightly overpredicts the actual shipment totals, primarily because the delay factors (from the Delayed Entry Program) obtained from the Recruiting Command assume no attrition, i.e. all signed contracts convert within the next 12 months to shipments. However, in reality there is a small attrition factor (estimated at about 1-3%) which can be directly incorporated in the model when such data becomes available. This shortcoming notwithstanding the results verify the descriptive ability of the model to both forecast contracts accurately and their conversion to shipments.

5.0 "VALIDATION" OF OPTIMIZATION MODEL FOR ALLOCATING BUDGETS

5.1 Scenario Analyzed

Consider next the "validation" of the allocation program which optimizes, within the budget constraints, the advertising and recruiter expenditures to minimize shortfalls. This comparison, much more complicated than the former one, had the following characteristics and assumptions:

i) The allocation model would be exercised at the regional level, of which there are 6 in the country, and by month for the CY77. Total dollars of advertising and the numbers of recruiters allocated would be the key decision variables compared. Hence the computerized model had a total of 144 decision variables, 2x12x6 (i.e. 2 types of expenditures, 12 months and 6 regions);

ii) In order to make the comparisons meaningful, the actual total budget expenditures for CY77 for advertising and recruiting were used as budget constraints in the computerized model; the recruiter expenditures also included his office space, travel, etc. The budget constraints for CY77 were $71.37 Million for recruiters and $16.07 Million for advertising.

iii) The objectives of the model were taken to meet the same set of total male, non prior service, monthly shipment quotas as was actually used in CY77. This total was 83,116. The model also utilized a constraint related to the total number of contracts signed in the year that must be HSG's; this number was 54,202 and was the actual number of HSG's signed in CY77. The model could not utilize any shipping quotas for HSG's because of a data limitation relating to the delayed entry program. In particular the delay factors $b_{j,v,k}$'s which measure the elapsed time between signing of the contract and shipment, have only been developed by the Recruiting Command for all contracts and are not available at this time for HSG contracts. Hence since there is no credible way to transform HSG contracts into HSG shipments, the quality constraint has been on total contracts. When the delay factors for HSG's become available the runs will be repeated with an additional quality constraint on shipments.

iv) The model takes into account in an explicit fashion various types of "initial conditions" with which the decision maker had to deal at the beginning of the horizon, i.e. those decisions or conditions that already existed or had been taken before the beginning of calendar year 1977 over which the decision maker has no control. In particular there was:

a) a certain "stock of advertising" and "recruiter stock" representing the dollars that had been put into advertising and recruiters over the past year. In other words the numbers of contracts in

January 1977 is dependent not only on the amount of advertising and the numbers of recruiters expended in January 1977, but also on the amounts spent in the previous months. These initial conditions lead to a slight modification in the enlistment prediction functions for the first few months of 1977. These modifications, together with the initial conditions, are included in Appendix C-1, and C-2.

b) Another important type of initial condition incorporated in the model deals with an estimate of the number of shipments that are already in the pipeline resulting from contracts signed in the previous year. In other words because of the Delayed Entry Program, certain fractions of the individuals who signed contracts throughout CY76 will actually ship in January 1977; the percent, e.g. for those that sign in December and ship in January, is actually about 15.5%. Similarly a certain portion of those signing contracts in November will actually ship to boot camp in July; this percent is 10.3%. Hence a substantial portion of the total shipment quota of 83,116 for calendar year 77 will actually be filled from contracts obtained in 1976. Utilizing the delay factors available, for calendar year 1977, it can be expected that a total of 22,851 or 27.5% of the total quota of 83,116, will actually be met from contracts already in the Delayed Entry and signed in 1976. The set of DEP delay factors used is in Appendix B and the schedule of the number of shipments in the pipeline is contained in the information presented in Appendix D.

c) Finally, there are two other environmental variables which capture the demographics of the regions; these are the numbers of HSG candidates in each region and its labor force. These have been inte-

grated to yield the set of $P_{i,j,k}$'s shown in Appendix A.

v) Finally terminal conditions for the exercising of the model were developed by applying the DEP delay factors of Appendix B to all contracts that were actually signed in 1977. This allows one to arrive at a set of shipment goals for 1978 for which the advertising/recruiter efforts in 1977 are accountable. In other words the efforts in CY77 must replace the DEP pool that was actually present at the end of CY77. This number is 13,430 and its distribution appears in Appendix D.

It is important to appreciate that if these terminal conditions were not imposed, then the results obtained might not be realistic. To see this, observe that if the quotas for December 1977 were being largely met from contracts obtained earlier in the year then the allocation logic, without the presence of the terminal conditions, might not allocate significant resources to December. In order to ensure that the model deals with the same long term or steady state requirements that the Recruiting Command actually faced, the terminal conditions need to be included. Hence, to summarize the allocations are only made for 1977, i.e. the horizon for decision making is 1 year, but that those decisions are driven by requirements over 2 years, i.e. 1977 and 1978.

vi) In exercising the model, it was assumed that once a recruiter had been assigned to a region, he could not be reassigned to another region within that year. Hence the mathematics of the model optimizes on the appropriate number of recruiters to have in each region, realizing that they cannot be shifted within the year horizon. This constraint which can be easily modified was included to reflect the realities and difficulties in transferring recruiters from one region to another over a short period of time.

5.2 Highlights of Results

For CY77, with the same sets of monthly quotas, with the same budget constraints (one for advertising and one for recruiting) and the same initial and terminal conditions, the results of the computerized allocation model, which has as its objective to minimize the sum of monthly shortfalls, were:

1) It produced in CY77 a total of 91,733 shipments (including those 22,891 in the delayed entry pool from contracts signed in CY76) at a total cost of \$81.762M or about \$5.7M (or 6.6%) less than was actually spent. Hence given that it was forced to operate within the same overall budget constraint and the same split between advertising and recruiting, it met the specified total quotas for CY77, the quality mix on contracts, as well as replacing the DEP pool present at the end of CY77. It spent almost the entire recruiting budget alloted i.e., \$71.26M of the \$71.37M alloted but only \$10.5M of the \$16.07M for advertising. Hence the relatively close fit to the total budget expended, given the limited accuracy (i.e., R^2's of .76) of the contract prediction fit lend support to the model's usefulness for operational purposes.

2) Finally, realizing that the actual yearly expenditures over the country agree quite well with the optimized yearly expenditures, it is of interest to compare the optimized and actual monthly results, both for contracts and for budget expenditures. (see Table 2) Observe then that the computerized model in a budget execution mode tends to spend somewhat less on advertising than was actually spent in the first six months and October and November, and then spends somewhat more than was actually spent in the summer months and in December. Note it meets the stated monthly quotas with slightly fewer contracts and shipments than actually occurred, partially because no attrition was assumed in the DEP program, and because the model in the above run did not allow excesses, relative to a given monthly quota, to compensate for shortfalls in another month. The model now has an option to input only a yearly quota and to have this compute, in effect, alternative optimal monthly quotas.

TABLE 2 ;

COMPARISONS OF MONTHLY DISTRIBUTION OF ACTUAL AND PREDICTED ALLOCATIONS UNDER SAME BUDGET CONSTRAINTS AND MONTHLY SHIPPING QUOTAS AS IN CY77

	Optimized Advertising Expenditures	Actual Advertising Expenditures	Optimized Number of Recruiter Man-Months	Actual Number of Recruiter Man-Months	Optimized Total Expenditures	Actual Total Expenditures	Optimized Flow of Shipments	Actual Flow of Shipments
Jan 77	$ .705M	$1.270M	3,363	3,220	$6.64 M	$6.956M	8,406	8,529
Feb 77	$ .745M	$1.189M	3,363	3,252	$6.68 M	$6.931M	6,299	6,833
March 77	$ .738M	$1.077M	3,363	3,267	$6.68 M	$6.846M	5,974	5,971
April 77	$ .611M	$ .836M	3,363	3,307	$6.55 M	$6.676M	4,745	5,051
May 77	$ .671M	$1.012M	3,363	3,352	$6.61 M	$6.931M	5,270	5,802
June 77	$ .858M	$1.961M	3,363	3,378	$6.80 M	$7.926M	10,143	10,203
July 77	$1.014M	$ .771M	3,363	3,403	$6.95 M	$6.780M	10,527	10,667
Aug 77	$1.219M	$ .707M	3,363	3,507	$7.16 M	$6.9 M	12,057	11,542
Sept 77	$1.437M	$ .755M	3,363	3,488	$7.38 M	$6.914M	10,730	11,189
Oct 77	$ .767M	$1.575M	3,363	3,430	$6.71 M	$7.632M	7,334	6,815
Nov 77	$ .972M	$1.173M	3,363	3,400	$6.91 M	$7.177M	5,999	5,830
Dec 77	$ .764M	$ .514M	3,363	3,406	$6.70 M	$6.528M	4,149	4,206
Total	$10.5 M	$16.07M	40,356	40,410	$81.762M	$87.44M	91,733	92,638

3) Next consider how the money was spent over the six regions by the model and in reality, remembering that the model was constrainted by the actual yearly budgets, one for advertising and one for recruiters, utilized in CY77. (see Table 3).

The major significant difference is for Region 500 (Chicago centered) where the optimized results were to cut back total expenditures there by about \$7.0M and give it to Region 100 and Region 300 (New England and Southeast).

6.0 SENSITIVITY EXCURSION OFF BASIC BUDGET EXECUTION RUN WITH ASSUMPTION THAT RECRUITER CAN DETERMINE DELAY FACTORS

This run was exactly similar to the budget execution run discussed earlier, i.e. some quotas, budget constraints, initial conditions, terminal conditions, etc. but with one exception: the delay factors in the DEP program were no longer treated as inputs but were part of the optimization process.

Hence the $b_{j,v,k}$'s were decision variables with the constraint $\sum_{v=0}^{12} b_{j,v,1} = 1$ for $j = 1,2,\ldots\ldots, J$

Under this assumption, namely that the recruiters have complete control over the lag between contract signing and shipping data, one could save another \$.25 million and still meet the stated quotas, the \$.25 million being in the advertising area. The major differences in the optimized delay factors and the empirical ones is: In December, only about 23% of all contracts are direct shipments (in the same month) but the model, if it had its choice, would prefer 97% of those signing contracts in December to direct ship. A similar situation holds for the months of March, April and May where only about 46% direct ship in reality but where the model would prefer almost all to direct ship. On the other hand, about 58% of those that sign contracts in August actually ship in the same month whereas the model would prefer most of those to enter the DEP program.

TABLE 3: COMPARISON OF ACTUAL GEOGRAPHICAL ALLOCATIONS VERSUS OPTIMIZED USING SAME BUDGET CONSTRAINTS AS EXPERIENCED IN CY77

	Optimized Advertising Expenditures	Actual Advertising Expenditures	Optimized Number of Recruiter Man Years for CY77	Actual Number of Recruiter Man Years	Total Expenditures from Model	Actual Total Expenditures	Number of Contracts From Model	Actual Number of Contracts
Region 100	$ 3.73M	$ 3.27	781	657	$20,278M	$17.19M	18,602	18,269
Region 300	$ 2.60M	$ 2.33	543	449	$14,110M	$11.84M	12,878	13,104
Region 400	$ 3.23M	$ 3.24	675	669	$17,533M	$17.42M	16,101	16,874
Region 500	$ 1.689	$ 3.17	353	538	$ 9,178M	$14.57M	8,710	12,903
Region 700	$ 1.99M	$ 1.80	417	437	$ 10.83M	$10.44M	9,971	10,687
Region 800	$ 2.84	$ 2.25	593	618	$ 15.41M	$15.35M	14,060	14,787
Total	$16.07M	$16.07M	3363	3368	$ 87.34M	$87.44M	80,321	86,624

APPENDIX A

CONTRACT PREDICTION PARAMETERS

The production function for total contracts from region i, month j were assumed to have the form

$$C_{i,j,k} = P_{i,j,k} \cdot X_{i,j}^{e_{0,k}} \cdot X_{i,j-1}^{e_{1,k}} \cdot X_{i,j-2}^{e_{2,k}} \cdot Y_{i,j}^{f_{0,k}} \cdot Y_{i,j-1}^{f_{1,k}} \cdot Y_{i,j-2}^{f_{2,k}}$$

where $X_{i,j}$ is the number of recruiters present in region i (in thousands), period j and $Y_{i,j}$ is the total dollar level of advertising (in millions of dollars) impacting region i in period j and $C_{i,j,k}$ is the total number of contracts of type k (in thousands) from region i, period j. Section 3 addresses this topic in detail. As has been mentioned earlier, $e_{0,1} = .29$, $e_{1,1} = .106$, $e_{2,1} = .039$ whereas $f_{0,1} = .125$, $f_{1,1} = .045$ and $f_{2,1} = .017$ where k=1 refers to all contracts as a whole. The numbers for HS6 contracts, i.e., K= 2, are $e_{0,2} = .391$, $e_{1,2} = .138$, $e_{2,2} = .049$, $f_{0,2} = .081$, $f_{1,2} = .029$ and $f_{2,2} = .01$. Hence a 1% change in the number of recruiters present in period j causes a .29% change in the total number of contracts signed in period j, whereas a 1% change in the amount of advertising expended 1 month earlier causes a .029% change in the number of HS6 contracts signed in the current period. The $P_{i,j,k}$'s vary by region and by month, reflecting the different numbers of QMA in the region, different propensities for enlistment, etc. and the seasonal nature of contracts. The factors for k=1, i.e. total contracts, follow below:

MATRIX OF $P_{i,j,1}$

		Region 100 (i=1)	300 (i=2)	400 (i=3)	500 (i=4)	700 (i=5)	800 (i=6)
(j=1)	January 77	2.41	2.1	2.26	1.73	1.88	2.15
(j=2)	February 77	2.46	2.14	2.27	1.77	1.93	2.2
(j=3)	March 77	2.68	2.33	2.54	1.94	2.1	2.41
(j=4)	April 77	2.07	1.8	1.96	1.5	1.63	1.86
(j=5)	May 77	1.89	1.65	1.79	1.63	1.48	1.7
(j=6)	June	2.37	2.07	2.24	2.53	1.87	2.14

		Region 100	300	400	500	700	800
(j=7)	July 77	2.61	2.28	2.46	1.9	2.05	2.34
(j=8)	August 77	2.83	2.47	2.67	2.05	2.22	2.55
(j=9)	September 77	2.74	2.40	2.6	1.99	2.17	2.49
(j=10)	October 77	1.95	1.71	1.86	1.42	1.55	1.75
(j=11)	November 77	2.25	1.97	2.14	1.64	1.77	2.01
(j=12)	December 77	2.54	2.2	2.39	1.84	2.0	2.3

We observe that Region 500 for each month has a fairly low yield for a given number of recruiters and advertising dollars whereas Region 100 has among the highest yield. This makes itself felt in that the fact that the allocations for Region 500 are less than for any of the regions and similarly more for Region 100. Similarly if we look at the effectiveness of a given amount of advertising on contract signing it appears to have the most impact in the months of March, June, July, August and September, and December, and the least effect in January and October. The factors for k=2, i.e. high school graduate contracts is as follows:

MATRIX OF $P_{i,j,2}$

		Region 100	300	400	500	700	800
(j=1)	January	1.56	1.47	1.5	1.07	1.18	1.51
(j=2)	February	1.54	1.45	1.45	1.06	1.16	1.49
(j=3)	March	1.61	1.52	1.56	1.11	1.22	1.57
(j=4)	April	1.27	1.20	1.23	.88	.96	1.23
(j=5)	May	1.19	1.13	1.15	1.07	.90	1.16
(j=6)	June	1.59	1.50	1.54	1.74	1.21	1.55
(j=7)	July	1.67	1.57	1.60	1.15	1.26	1.62
(j=8)	August	1.76	1.66	1.7	1.21	1.33	1.71
(j=9)	September	1.57	1.49	1.53	1.09	1.2	1.54
(j=10)	October	1.16	1.1	1.13	.81	.89	1.12
(j=11)	November	1.27	1.2	1.23	.88	.96	1.23
(j=12)	December	1.36	1.27	1.30	.93	1.934	1.33

APPENDIX B

DELAY FACTORS IN DEP PROGRAM FOR ALL CONTRACTS

DATE	0	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10	+11	+1
Jan	.5188	.1389	.0559	.0327	.0236	.0663	.0568	.0561	.0351	.0219	.0011	.0005	.0
Feb	.4613	.1273	.0537	.0472	.0657	.0635	.0762	.0576	.0248	.0166	.0014	.0007	.0
Mar	.4374	.1171	.0705	.0806	.0667	.0708	.0574	.0468	.0305	.0184	.0024	.0007	.0
Apr	.4451	.1183	.1015	.0702	.0762	.0527	.0543	.0325	.0184	.0267	.0019	.0016	.0
May	.4825	.1655	.0875	.0742	.0536	.0511	.0327	.0184	.0186	.0125	.0008	.0007	.0
Jun	.5598	.1452	.0936	.0568	.0476	.0366	.0175	.0144	.0137	.0064	.0010	.0009	.0
Jul	.5995	.1635	.0571	.0556	.0468	.0197	.0228	.0106	.0080	.0049	.0006	.0086	.0
Aug	.5824	.1408	.0918	.0584	.0396	.0290	.0168	.0112	.0057	.0053	.0113	.0034	.0
Sep	.5912	.1258	.0700	.0386	.0434	.0246	.0125	.0063	.0045	.0702	.0071	.0032	.0
Oct	.5700	.1313	.0436	.0553	.0247	.0208	.0093	.0078	.0586	.0636	.0084	.0043	.0
Nov	.4300	.0886	.1018	.0421	.0263	.0149	.0152	.0614	.1028	.1003	.0107	.0045	.0
Dec	.2262	.1552	.0677	.0487	.0274	.0249	.1042	.0837	.1214	.1118	.0169	.0060	.0

<u>Example:</u>

As an example, 51.88% of those signing enlistment contracts in January of a given year are assumed to ship in the same month whereas only 22.62% of those signing in December will direct ship. Also for those signing contracts in January, about 6.63% will ship 5 months later, i.e. in June.

APPENDIX C - 1

INITIAL CONDITIONS RELATED TO "STOCKS" OF ADVERTISING/RECRUITERS

From Last Half of 1976

Advertising Expenditures/Number of Recruiters

	July 76	Aug 76	Sept 76	Oct 76	Nov. 76	Dec. 76
Region 100	$208K/613R	$202/627R	$146K/632R	$276K/615R	$345K/616R	$230K/609R
Region 300	$120K/403R	$121/408R	$94/411R	$118/407R	$162K/416R	$93K/389R
Region 400	$183K/633R	$634/178R	$651/150R	$645/241R	$654/298K	$651/199KR
Region 500	$150K/506R	$140/504R	$124/506R	$177/503R	$193/500R	$163K/495R
Region 700	$108K/422R	$106/436R	$76 /439R	$150/427R	$201/437R	$128K/434R
Region 800	$157K/583R	$153/596R	$112/593R	$103/584R	$137/585R	$ 70K/599R
TOTAL	$926K/3160KR	$899/3124R	$702/3232R	$1064/3181R	$1336/3208R	$883K/3127R

Illustration: In December, 1976 there were 609 Recruiters in Region 100 and $230,000 of advertising was expended there.

APPENDIX C - 2

MODIFICATIONS TO ENLISTMENT PREDICTION CONSTRAINTS ($P_{i,j,k}$'s) AS RESULT OF INITIAL CONDITIONS

	Region 100	Region 300	Region 400	Region 500	Region 700	Region 800
January 77						
Original prediction constant ($p_{i,1,1}$) for total contracts	2.408	2.096	2.264	1.73	1.88	2.15
Modified prediction constant for total contracts as result of initial conditions	2.061	1.60	1.94	1.40	1.49	1.71
Original prediction constant ($p_{i,1,2}$) for HSG contract	1.562	1.47	1.501	1.07	1.176	1.509
Modified prediction constant for HSG contracts as result of initial condition	1.350	1.13	1.306	.88	.933	1.24
February 77						
Original prediction constant ($p_{i,2,1}$) for total contracts	2.462	2.142	2.271	1.77	1.93	2.20
Modified prediction constant for total contracts	2.35	1.98	2.173	1.58	1.80	2.059
Original prediction constant ($p_{i,2,2}$) for HSG contracts	1.540	1.448	1.454	1.057	1.164	1.489
Modified prediction constant for HSG contracts	1.480	1.351	1.401	1.021	1.095	1.414

APPENDIX D

Shipment Quotas In Effect in CY77 and Terminal Conditions for 1978

	Quotas for 77	Net Quotas for 77 After Taking Out Pipeline from 76	Terminal Condition Quotas Used for 78 (from Contracts in 1977)
January	7,415	2,321	3,273
February	5,645	3,349	1,508
March	5,164	3,601	962
April	4,560	3,751	498
May	4.932	3,840	424
June	8,339	4,829	2,226
July	9,738	6,822	1,764
August	11,124	8,092	1,637
September	10,730	8,737	887
October	6,279	5,944	159
November	5,041	4.929	51
December	4,149	4,050	41
Total	83,117	60,265	13,430

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The report summarizes model building and validation efforts directed towards predicting and optimizing of Navy's advertising and recruiter efforts. Both contract prediction results are shown as well as budget execution/budget generation models and results. The models were exercised using CY77 data and the model's results compared with actual allocations.

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